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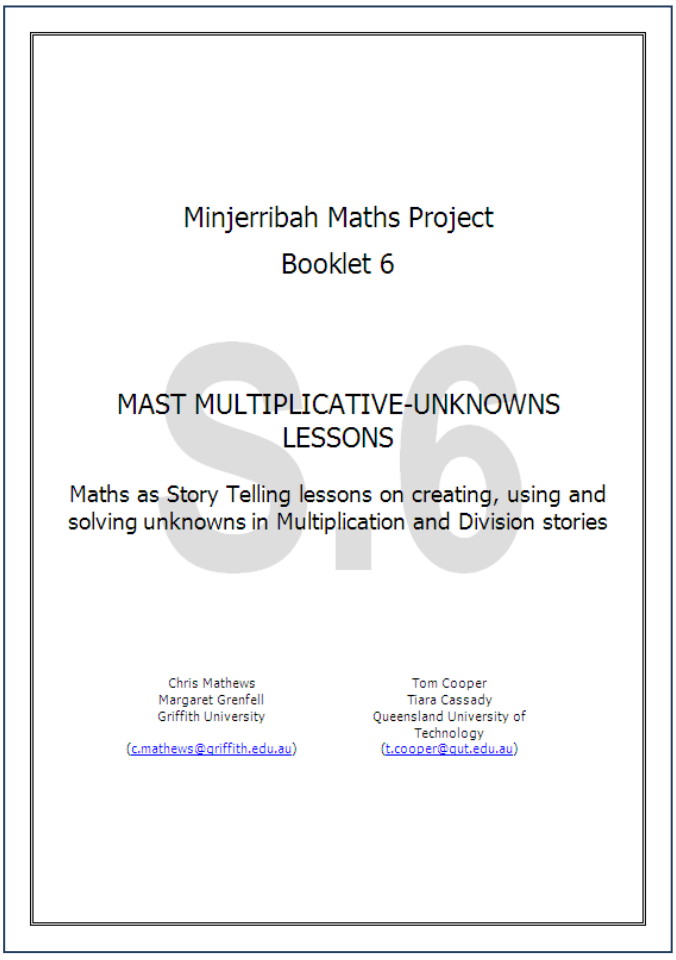
Minjerribah Maths Project

MAST Multiplicative-Unknown Lessons

Booklet S.6: creating, using and solving unknowns in Multiplication and Division stories



Minjerribah Maths Project

Booklet 6

S.6

MAST MULTIPLICATIVE-UNKNOWNS LESSONS

Maths as Story Telling lessons on creating, using and solving unknowns in Multiplication and Division stories

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YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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CRICOS No. 00213J

This booklet was developed as part of a project which ran from 2006–2008 and was funded by an Australian Research Council Discovery Indigenous grant, DI0668328: *Indigenous world view, algebra pedagogy and improving Indigenous performance in secondary mathematics*.

Queensland University of Technology

MINJERRIBAH MATHS PROJECT

MAST

MAST MULTIPLICATIVE-UNKNOWN LESSONS

BOOKLET S.6

CREATING, USING AND SOLVING UNKNOWNS IN MULTIPLICATION AND DIVISION STORIES

VERSION 1: 2007

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BACKGROUND

Minjerribah Maths Project

The Minjerribah Maths Project was a research study to find effective ways to teach Indigenous students mathematics, and for Indigenous students to learn mathematics. It realised that effective mathematics teaching is crucial for Indigenous students' futures as mathematics performance can determine employment and life chances. It endeavoured to find ways that will encourage and enable more Indigenous students to undertake mathematics subjects past Year 10 that lead to mathematics-based jobs.

Some educators argue that Indigenous students learn mathematics best through concrete "hands-on" tasks, others by visual and spatial methods rather than verbal, and still others by observation and non-verbal communication. However, these findings may be an artefact of Indigenous students being taught in Standard Australian English with which they may not have the words to describe many mathematical ideas and the words they have may be ambiguous. It is important to recognise that Indigenous students come from a diverse social and cultural background and investigations into Indigenous education should take this into consideration. Indigenous people also have common experiences, which can be reflected upon to suggest ways forward.

There is evidence that school programs can dramatically improve Indigenous learning outcomes if they reinforce pride in Indigenous identity and culture, encourage attendance, highlight the capacity of Indigenous students to succeed in mathematics, challenge and expect students to perform, provide a relevant educational context in which there is Indigenous leadership, and contextualising instruction into Indigenous culture. However, the majority of teachers of Indigenous students are non-Indigenous with little understanding of Indigenous culture and these non-Indigenous teachers can have difficulties with contextualisation and reject it in favour of familiar Eurocentric approaches. Thus, there is a need to build productive partnerships between non-Indigenous teachers and the Indigenous teacher assistants employed from the community to assist them and the Indigenous community itself.

There is also some evidence that Indigenous students tend to be holistic, learners, a learning style that appreciates overviews of subjects and conscious linking of ideas and should appreciate algebraic structure. Thus, algebra could be the basis for Indigenous mathematics learning. This position is interesting because algebra is the basis of many high status professions. It is also based on generalising pattern and structure, skills with which Indigenous students may have an affinity because their culture contains components (e.g., kinship systems) that are pattern-based and which may lead to strong abilities to see pattern and structure. Finally, algebra was the vehicle which enabled the first Indigenous PhD in mathematics to understand mathematics. As he reminisced:

> When reflecting back on my education, my interest in mathematics started when I began to learn about algebra in my first year of high school. … For me, algebra made mathematics simple because I could see the pattern and structure or the generalisation of algebra much clearer than the detail of arithmetic.

Therefore, the Minjerribah Maths Project was set up to answer the following questions. *Can we improve achievement and retention in Indigenous mathematics by refocusing mathematics teaching onto the pattern and structure that underlies algebra? In doing this, are there Indigenous perspectives and knowledges we can use? Can we at the same time provide a positive self-image of Indigenous students?*

The project's focus was to put Indigenous contexts into mathematics teaching and learning (making Indigenous peoples and their culture visible in mathematics instruction) and to integrate the teaching of arithmetic and algebra (developing the reasoning behind the rules of arithmetic, while teaching arithmetic, so that these can be extended to the rules of algebra). The overall aim is to improve Indigenous students' mathematics education so they can achieve in formal abstract algebra and move into high status mathematics subjects.

This project was undertaken through action-research collaboration with Dunwich State School teachers by putting into practice processes to improve and sustain these enhanced Indigenous mathematics outcomes. The research is qualitative and interpretive and aims to address Indigenous mathematics-education questions in ways that give sustained beneficial outcomes for Indigenous people. It is based

on the following analysis of the present situation with regard to Indigenous mathematics teaching and learning.

The Minjerribah Maths Project was a collaboration between Griffith University and Queensland University of Technology with Dr Chris Matthews from Griffith University as coordinator. The researchers involved in the project were:

- Dr Chris Matthews, Coordinator and Principal Researcher, Centre for Environmental Systems Research, Griffith University;
- Professor Tom Cooper, Researcher, Centre for Learning Innovation, QUT;
- Ms Margaret Grenfell, Research Assistant, Centre for Environmental Systems Research, Griffith University;
- Ms Tiara Cassady, Research Assistant, Centre for Learning Innovation, QUT.
- Mr Todd Phillips, Research Assistant, Centre for Environmental Systems Research, Griffith University.
- Ms Ashlee Surha, Research Assistant, Centre for Environmental Systems Research, Griffith University.

MAST (Maths as Story Telling) pedagogy

MAST is the first product developed for the Minjerribah Maths Project. It is an attempt to work from the storytelling world of the Indigenous student through to the formal world of algebra by experiences with the creation of symbols that have personal meaning. The storytelling starts with simple arithmetic but moves quickly to algebraic thinking. It enables Indigenous students to bring their everyday world of symbols into mathematics.

It is an answer to the dilemma of contextualising the teaching and learning of algebra. It focuses on representing mathematical equations as stories which leads to contextualising of mathematical symbols. It is an approach to symbolisation based on students creating and using their own symbols, drawn from their socio-cultural background, to describe these stories as a precursor to working with the accepted mathematics symbols. It utilises the current knowledge of the Indigenous student, which is drawn from their world, such as art, dancing, sport or driving, as a starting point for building understanding of arithmetic symbolism in a way that can be easily extended to algebraic symbolism. The approach has five steps. These steps are explained for addition. Obviously, the other three operations could be similarly undertaken.

Step 1. Students explore the meaning of *symbols* and how symbols can be assembled to tell and create a story. This is initially done by looking at symbols in Indigenous situations (e.g., exploring and understanding symbols in paintings) and then creating and interpreting symbols for simple actions (e.g. walking to and sitting in a desk).

Step 2. Students explore *simple addition story* by acting it out as a story (e.g. two groups of people joining each other). A discussion is then generated to identify the story elements such as the different groups of people and the action (the joining of the two groups) and the consequences of the action (the result of the joining).

Step 3. Students *create their own symbols* to represent the story. This step could be done in a freestyle manner; however, we have opted to take a more structured approach by using concrete materials (which are familiar to the students) to represent the objects (or people) in the story. The story is then created by allowing the students to construct the two groups of people with the concrete materials and construct their own symbol for "joining two groups" and lay this out to represent the action (or history) of the story. In a similar fashion, the students then construct their own symbol for "resulting in" or "same as" to tell the story of what happens after this action has taken place. Figure 1 gives an example of an addition story that was constructed by a student in Year 2.

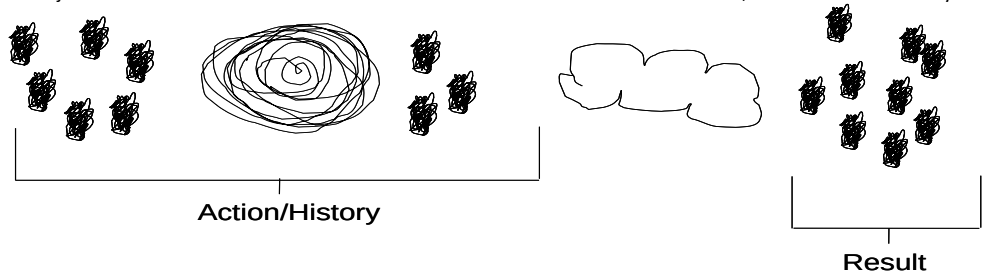


Figure 1. A Year 2 student's representation of the addition story 6 + 3 = 9.

Step 4. Students share their symbol systems with the group and any addition meanings their symbols may have. For example, in Figure 1, the student's "joining" symbol was a vortex that sucked the two groups together. The teacher then selects one of the symbol systems for all the students to use to represent a new addition story. This step is important to accustom students to writing within different symbol systems and to develop a standard classroom symbol system.

Step 5. Students *modify the story* (a key step in introducing algebraic ideas) under direction of the teacher. For example, the teacher takes an object from the action part of the story (see Figure 1), asks whether the story still makes sense (normally elicits a resounding "No"), and then challenges the students by asking them to find different strategies for the story to make sense again. There are four possibilities: (1) putting the object back in its original group, (2) putting the object in the other group on the action side, (3) adding another action (plus 1) to the action side, and (4) taking an object away from the result side. The first three strategies introduce the notion of compensation and equivalence of expression, while the fourth strategy introduces the balance rule (equivalence of equations). At this step, students should be encouraged to play with the story, guided by the teacher, to reinforce these algebraic notions.

Step 6. Students explore the meaning of *unknown* under direction of the teacher. For example, the teacher sets an example with an unknown (e.g. John bought a pie for $3 and an ice cream and he spent $7). The teacher asks the students to represent this without working out the value of the ice cream. Students invent a symbol for unknown and use it in stories with unknowns. Then the students are challenged to solve for unknowns using the balance rule. They have to first determine the operations to leave the unknown on its own. Thus, begins solutions to unknowns in linear equations.

Mathematics behind MAST

The MAST pedagogy is a way of introducing concepts, principles and unknowns for the four operations. The mathematics behind the activities in the booklets is now discussed.

Symbols and concepts (Booklets 1 and 4). The symbols for the four operations and equals are + addition, - subtraction, x multiplication, ÷ division, and = equals. Numbers and these symbols make up expressions (a number sentence without equals, such as 3+4 or 6x7-3) and equations (a number sentence that has an equals sign, such as 3+4=7 or 40-1=6x7-3). The concepts of the operations are complex and cover many situations. The best meanings for the four operations are as follows.

(1) Addition and subtraction are when situations involve joining to make a total or separating a total into parts – addition is when the parts are known and the total is unknown, and subtraction is when the total and one unknown is known and the other part is unknown. For example, in the story *I went to the bank and took out $7.983; this left $5,205 in the bank; how much did I have to start from?*, the operation is addition because $7,983 and $5,205 are parts and the amount at the start is the total and is the unknown (even though the action and the language is "take-away").

(2) Multiplication and division are when situations involve combining equal groups to make a total and separating the total into equal groups – multiplication is when the number of groups and the number in the group is known and the total is unknown, and division is when the total is known and one of the number of groups or number in each group is unknown. For example, in the story *There are 8 times as many oranges as apples; there are 56 oranges; how many apples?*, the operation is division because 8 is the number of groups, 56 is the total and the number of apples is an unknown group (even though the action and language is "times").

However, for these booklets, the following initial simpler (and incomplete) meanings are used:

Addition - joining Subtraction - take-away,

Multiplication - combining equal groups Division - sharing or breaking into groups equally.

The idea in Booklets 1 and 4 (addition and subtraction in 1, and multiplication and division in 4) is that mathematics symbols are a way of telling stories of everyday life with these meanings.

Principles (Booklets 2 and 5). An important component of algebra is understanding when these two things do not change (i.e., are equivalent or invariant) because in this case we can construct arguments where we replace the expression or equation with an equivalent one and find answers and solution to unknowns, such as 6x7-3 = 42-3 = 39 and 6*x*-3 = 39 is the same as 6*x*-3+3 = 39+3 is the same as 6*x* = 42 is the same as 6*x*/6 = 42/7 which is the same as *x*=7. Thus it is important in both arithmetic and algebra to teach when expressions and equations are equivalent.

Two important principles or rules for equivalence are compensation for expressions and balance for equations. Compensation is the set of principles/rules that mean that an expression remains the same if one number is changed. Balance is the set of principles where the equation remains the same after something is done to one side of the equation. For the four operations, the principles are as follows.

(1) The compensation principle for addition is "to do the opposite". In the example, 3 + 4, adding a 2 to the 3 is compensated by subtracting a 2 from the 4 (i.e. 5 + 2 is the same as 3 + 4). Similarly, subtracting 2 from the three is compensated by adding 2 to the 4 (i.e. 1 + 6). These sets of operations can also be done to the 4, which will always result in doing the opposite operation to keep the expression equivalent using the compensation principle.

(2) The compensation principle for subtraction is "to do the same" (the opposite to the principle for addition). Adding/subtracting 2 to or from the 7 in example 7-3 is compensated by adding/subtracting (same operation) 2 to or from the 3 making 9-5 and 5-1 both of which give the same answer as 7-3. Similarly, adding/subtracting 2 to or from the 3 in example 7-3 is compensated by adding/subtracting 2 to or from the 7.

(3) The compensation principles for multiplication is "to do the opposite" (the same as addition and the opposite to subtraction). Multiplying/dividing the 6 by 2 in example 6x4 is compensated by dividing/multiplying (opposite operation) the 4 by 2 making 12x1 and 3x4 both of which give the same answer as 6x4. Similarly, multiplying/dividing the 4 by 2 in 6x4 is compensated by dividing/multiplying the 6 by 2.

(4) The compensation principle for division is "to do the same" (the opposite to addition and multiplication and the same as subtraction). Multiplying/dividing the 18 by 3 in example18÷6, is compensated by multiplying/dividing the 6 by 3 to make 54÷18 and 6÷2 both of which give the same answer as 18÷6. Similarly, multiplying/dividing the 6 by 2 in example 18÷6 is compensated by multiplying/dividing the 18 by 2.

(5) The balance principle for addition is "to do the same to both sides of the equals sign". Adding/subtracting 2 to or from the 3 in example 3+4=7 is balanced by adding/subtracting 2 to and from the 7 on the other side making 5+4=9 or 1+6=5, both of which are equivalent to 3+4=7. Similarly, addiing/subtracting 2 to or from the 4 in example 3+4=7 is balanced by adding/subtracting 2 to and from the 7 on the other side.

(6) The balance principle for subtraction is "to do the same to both sides of the equals sign" (the same principle as for addition) but is complex when the second number is involved. Adding/subtracting 2 to the 7 in example 7-3=4 is balanced by adding/subtracting (same operation) 2 to the 4 on the other side making 9-3=6 and 5-3=2 both of which are equivalent to 7-3=4. However, adding/subtracting 2 to the 3 in 7-3=4 is adding/subtracting to a take-away so is really subtracting/adding 2 to 7-3. This means that it is balanced by subtracting/adding (opposite to what was done to the 3 but the same as effect on the 7-3) 2 to the 4 on the other side making 7-5=2 and 7-1=6 both of which are equivalent to 7-3=4.

(7) The balance principle for multiplication is to "do the same thing to both sides of the equals sign" (the same principle as addition and subtraction – but without complexity for the second number – unlike subtraction). Multiplying/dividing the 6 by 2 in example 6x4=24 is balanced by

multiplying/dividing (same operation) the 24 on the other side making 12x4=48 and 3x4=12 both of which are equivalent to 6x4=24. Similarly multiplying/dividing the 4 by 2 in 6x4=24 is balanced by multiplying the 24 by 2 on the other side.

(8) The balance principle for division is "to do the same thing for both sides of the equals sign" (but with complexity for the second number – the same as subtraction and different in this aspect from addition and subtraction). Multiplying/dividing the 18 by 3 in example 18÷6=3 is balanced by multiplying/dividing (same operation) the 3 on the other side giving 54÷6=9 and 6÷6=1 both of which are equivalent to 18÷6=3. However, multiplying/dividing the 6 by 3 in example 18÷6=3 is multiplying/dividing a division (the number sharing or the size of the group) so is really dividing/multiplying 18÷6 by 3. This means that it is balanced by dividing/multiplying (the opposite to what was done to the 6 but the same as what was done to the 18÷6) by 3 the 3 on the other side making 18÷2=9 and 18÷18=1 both of which are equivalent to 18÷6=3.

With respect to both compensation and balance, addition and multiplication act similarly, and subtraction and division also act similarly but have differences from addition and subtraction. With respect to compensation, addition and multiplication "do the opposite", while subtraction and division "do the same". With respect to balance, all operations "do the same to both sides", but subtraction and division are complex with respect to the second number where "do the opposite" appears to hold. However, the actions to both sides are "the same".

Unknowns (Booklets 3 and 6). When not all numbers are known in a situation, this not-known number has to be represented by a new symbol called a variable or unknown. In formal mathematics, this is represented by a letter. Informally, it can be represented by a box or question mark.

The balance principle can be used to find the unknown. The steps are as follows: (1) the operation (or operations are worked out that will leave the unknown alone on one side; and (2) these operations are balanced on the other side to find the value of the unknown. For example:

$y + 3 = 11$	$y \div 4 = 3$	$5 \times y + 6 = 21$
change the 3 to 0	change the 4 to 1	subtract 6
by subtracting 3	by multiplying by 4	$5 \times y = 21 - 6 = 15$
$y = 11 - 3$	$y = 3 \times 4$	divide by 5
$y = 8$	$y = 12$	$y = 15 \div 5 = 3$

Approach to teaching MAST

A crucial component in any successful program to improve Indigenous students' learning outcomes in mathematics is **high teacher expectations** for these students' learning.

The MAST program has been designed to take account of the strengths of Indigenous students in terms of cultural and social background and learning style. This should be made evident to the Indigenous students at the beginning of each booklet.

Be direct. State that the mathematics they are to do now is different to the mathematics that they have done before – state that it is designed for Goori or Murri students but all students can excel in the new approach. The approach has

- never been trialled anywhere else in the world and you will be the first to try it out;
- it focuses on being creative, to think about mathematics in a different way; and
- to relate mathematics to the world around us.

Be firm in your conviction that all students will be able to do it and, in fact, will thrive and prosper.

ABOUT THIS BOOKLET

Focus of lessons

The four lessons in this booklet are the third level lessons for MAST. They cover Step 6 for multiplication and division. They focus on introducing the idea of unknowns and using balance principle to solve for unknowns. Similar to booklets 1 to 5, the lessons relate to different representations:

(1) Unknowns in multiplication and division

(2) Real life situations

(3) Acting out these situations

(4) Symbol constructed by students and,

(5) The formal symbols of mathematics

These Lessons are suitable from Year 5 onwards and can be used into junior secondary school. They could also be used earlier. The lessons attempt to work with unknowns as follows.

(1) **Lesson 1** introduces the concept of "unknown" or "any number" and utilises a created symbol for unknown in multiplication and division expressions and equations. Students create their own symbol for the unknown.

(2) **Lesson2** introduces balance principle for multiplication with unknown and uses this balance principle to solve for unknown in multiplication stories. Students use the balance principle to ensure equations remain balanced when they change the equation so that the unknown is alone on one side.

(3) **Lesson 3** introduces balance principle for division with unknown and uses balances principle to solve for unknown in uses balance principle to solve for unknown in division stories. Students use the balance principle to ensure equations remain balanced when they change the equation so they the unknown is alone on one side.

(4) **Lesson 4** relates invented symbols to formal mathematics symbols and solve for unknown in formal multiplication and division equations. Includes equations with more than one operation.

This completes the 6 MAST booklets where Booklets 1–3 focus on addition and subtraction and Booklets 4-6 focus on multiplication and division. Booklets 1 and 4 focus on concept, Booklets 2 and 5 focus on principle and Booklets 3 and 6 focus on unknowns and solving for unknowns.

Hints for teaching

The Lessons give detailed directions with respect to the Lessons. The first rule is that THESE DIRECTIONS DO NOT HAVE TO BE FOLLOWED. Develop your own approach to the Lessons – mix and match from different Lessons (and booklets) – spend more time on certain ideas, and give a lot more reinforcement and practice examples than in these lessons.

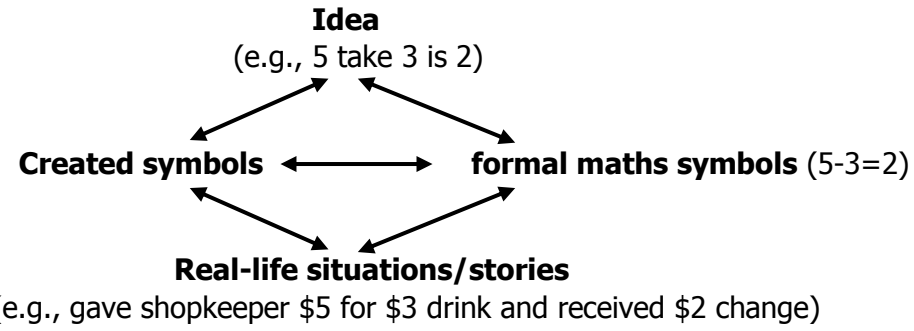
The best way to operate is to keep in mind where you want to go and let the students' responses to your earlier teaching direct your later teaching. There can be great merit even in integrating ideas from different booklets.

However, in our few trials, some things have been found useful.

(1) Drawing symbols for a simple task such as a child walking to a desk really works as an introduction to the maths drawings. It is useful to focus on objects and actions and get the students to think of the drawing being like a cartoon showing a series of symbols to tell a story.

(2) If teachers have blu tack and magnetic counters, they can set up a white board so that students can stick their A5 drawings on the board between counters to discuss their inventions.

(3) Always move the Lessons in <u>both directions</u> and then in <u>all directions</u> – from idea to created symbols and from created symbols to idea, from created symbols to story and story to created symbols, and from idea to created symbols and created symbols to idea. Act out things as well as talk about things. Overall, lessons will be trying to build all these relationships:



(4) Relate to the symbols in all ways too. A good sequence appears to be:

- Get students to draw their own symbols for your stories and then to make up their own stories for their own symbols.
- Get students to draw stories in other students' symbols and then to make up stories for other students' symbols
- Get students to draw stories in formal symbols and then to make up stories for formal symbols

(5) Take every chance to look at similarities and differences between addition and subtraction (e.g., different symbols for add and subtract but the same symbol for makes, gives or equals).

(6) Introduce formal mathematics symbols in a similar way that would get students to use other students' created symbols. Watch that the introduction of formal symbols does not change students' focus to "getting answers" rather than discussing/inventing rules.

MAST MULTIPLICATIVE UNKNOWN: LESSON 1

Objectives:

- To introduce concept of "unknown" or "any number".
- To utilise created symbols of unknown in multiplication and division expressions and equations.

Materials:

- Counters (preferably natural objects) for students.
- Magnetic counters and blu tack for the teacher
- A4 and A5 sheets
- Board set up with:

Counters | blu tack | counters | blu tack | counters

- Calculators, Worksheet 1 & 2

Language: Symbol, story, multiplication, division, unknown, equals, representation

What teacher does:

1. Recap lessons on creating symbols for multiplication and division.

   Hand out white and coloured A5 papers and counters and ask students to draw symbols for multiplication and division and equals on A5 white paper.

2. Ask students to represent story: "My mother gave me 5 vouchers worth $3 each. Now I have $15".

   State that sometimes, we do not know or cannot remember what the number is, for example: "My mother gave me vouchers worth $3 each. How much do I have?"

   State that in this situation, the amount I have in my pocket could be any number of vouchers so I need a new symbol that stands for "any number" or "unknown". Ask students to make up such a symbol on coloured A5 paper. Organise for students to share and discuss symbols.

   Ask students to use created symbols to construct the example. Organise students to share what they produce.

What children do:

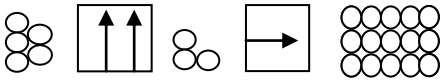
Discuss previous lessons and recall what was covered.

Draw symbols for multiplication and division and equals on white A5 paper, e.g.

 multiplication  division  equals

(These are examples only. The students will make up their own symbols).

Draw story: 5 multiply 3 is 15

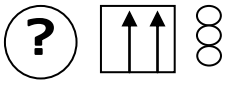


Discuss idea of "unknown" or something that could be "any number".

Create symbol for unknown e.g. 

(This is an example only. The students will make up their own symbols).

Share symbol with other students and discuss what makes a good symbol.

Construct story, "unknown multiply 3; with <u>own</u> symbols e.g.: 

Share stories and symbols with other students.

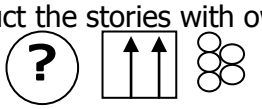
3. Ask students to use created symbols for other stories with unknowns:

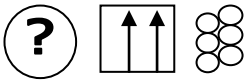
   (a) I rode a bike around the 5km track many times. How far did I ride?

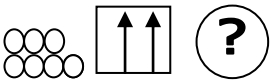
   (b) I bought chocolates for all the visitors. The chocolates cost $6 each. How much did I spend?

   (c) The 7 players scored the same. How much did the players score together?

Construct the stories with own symbols:

(a) e.g. 

(b) e.g. 

(c) e.g. 

4. Repeat steps 2 and 3 above for division.

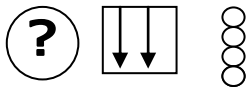
   Ask students to use created symbols for the following stories:

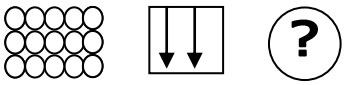
   (a) "I shared the money amongst 4 people. How much money did each person get?

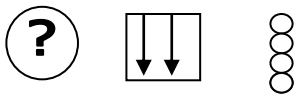
   (b) "I shared $15 amongst the children. How much money did each person get?

   (c) "The team ran a relay race. There were 4 people in the team. How far did each member run?"

Construct stories with own symbols:

(a) e.g. 

(b) e.g. 

(c) e.g. 

5. State that sometimes we know the answer when we have an unknown and can use equals.

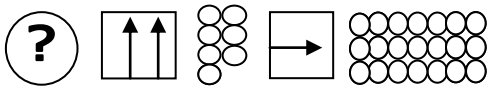
   Ask students to used created symbols for these stories:

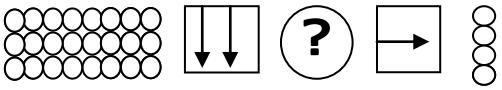
   (a) "John bought $7 tickets. He spent $21."

   (b) "Jan shared $24 amongst the people. Each person got $4.

   Get students to share their stories and symbols.

Construct stories with own symbols:

(a) 

(b) 

6. Direct students to complete Worksheet 1. Lead discussion of answers.

Complete Worksheet 1. Discuss answers.

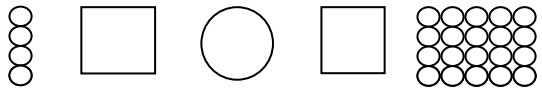
Evaluation:

- Students engage and seem able to construct stories with unknowns.
- Students create an "unknown" symbol and understand its meaning.
- Students successfully complete Worksheet 1.

MAST MULTIPLICATIVE UNKNOWN 1: WORKSHEET 1

Student name:	**School/class**:

Fill in the missing sections. Use your own symbols. We have done the first one for you.

Story	Action	Symbols
Example: Mary baked 4 batches of cakes. This made 20 cakes.	4 multiply unknown is 20	
1. Sam shared $18 amongst his friends. Each got $3.	18 divide unknown is 3	
2. Dad gave each child $6. He gave away $24.	Unknown multiply 6 is 24	
3. Jack bought 6 lunches with the money. Each lunch costs $5.		
4.	4 multiply unknown is 16	
5.	30 divide unknown is 15	
6.	Unknown multiply 7 is 21	
*7. I bought a $3 pie for each friend plus a $7 pizza. I spent $22.		

MAST MULTIPLICATIVE UNKNOWN: LESSON 2

Objectives:

- To introduce balance principles to <u>multiplication</u> with unknown.
- To use balance principle to solve for unknown in multiplication stories.

Materials:

- Counters (preferably natural objects) for students
- Magnetic counters and blu tack for teacher
- A5 sheets
- Board set up with:

Counters | blu tack | counters | blu tack | counters

- Calculators, Worksheet 1

Language: expression, equation, multiplication, division, unknown, representation, balance, solving for unknown

What teacher does:

1. Recap last lesson. Go through example. Ask students to construct story: "I bought 7 books. I paid $28.

2. Ask if anyone remembers the balance principle. Do an example. Ask students to construct: "I bought 6 games for 4 each. I spent $24".

   Ask students to balance the story if I divided the 4 by 2.

   Repeat the process for: "I gave Jenny 8 cheques for $2. I gave away 16".

   Ask students to balance the story if I increase the $2 by multiplying by 2.

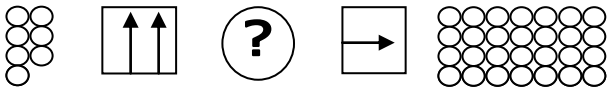
   Discuss answers given by students. Discuss what balance means (i.e. what is done on one side of equals, is done to the other).

3. Show how balance acts with unknowns. Use the example: "I drove to Amity Point many times. The trip to Amity Point was 5km. In total I drove 20km.

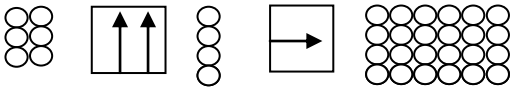
   Multiply the 5 by 2 and balance the story.

What children do:

Discuss previous lesson. Construct story with own symbol, e.g.

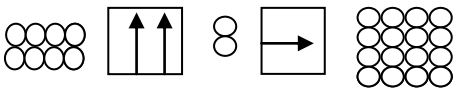


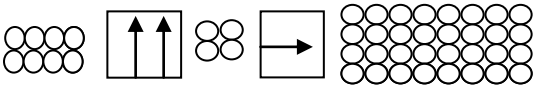
Discuss balance principle.
Construct addition example with own symbols: e.g.



Divide 4 by 2,

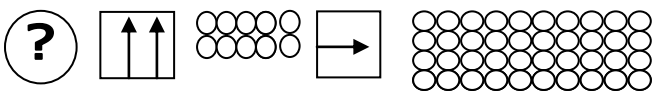
(dividing 4 by 2 means dividing 24 by 2)
Construct new example and balance the change:





(multiplying 2 by 2 means multiplying 16 by 2 = 32)

Construct story, change and balance:



(multiplying 5 by 2 means multiplying 20 by 2)

4. State that there is a way to work out unknowns by using the balance principle.

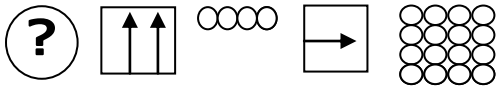
   Ask students to construct this multiplication example with an unknown: "I bought tickets for $4 each. I spent $16."

   Ask: *What change to the story will leave only the unknown on the left-hand-side of the equals?*

   Lead discussion to see that if the 4 is reduced to one, then the unknown would be all that is left because the unknown ones is unknown.

   Lead discussion that if balance for $4 being changed to one, which is dividing by 4, this will mean that unknown is what remains.

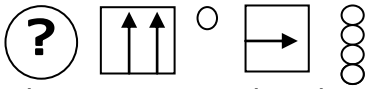
Construct the multiplication unknown example:



Share stories and symbols.

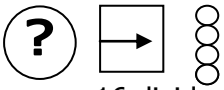
Discuss how could only have unknown on left-hand-side of equals.

Balance when spend $1 on games:



($1 on tickets means spending this divided by 4)

Understand this means:



(Spending $1 on games means 16 divide 4 is the number of tickets).

5. Look at another example: "The 5 players all scored the same number of points. Together they scored 15 points".

   Ask: *What we change to only have the unknown on left-hand side of equals?* Lead discussion to get students to see that we divide by 5.

   Discuss student's answers to the balanced story. Ask: *Who knows a rule for finding the unknown in a multiplication story?*

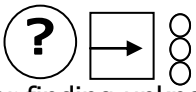
   Discuss rules. Ensure students realise you have to divide by the number which multiplies with the unknown.

Students construct:

Discuss how to get  on own in addition, multiplication.

When see it is to divide the 5 by 5, do this and balance.

Which means:



Discuss rules for finding unknowns in multiplication stories.

Direct students to complete Worksheet 1. Lead discussion of answers. Direct students to put in numbers when they are large and use calculators.

Ensure student know Alan's symbols.

Complete Worksheet 1. Discuss answers.

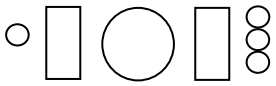
Evaluation:

- Students engage and offer opinions.
- Students understand how the balance rule finds the unknown for multiplication.
- Students successfully complete Worksheet 2.

MAST MULTIPLICATIVE UNKNOWN 2: WORKSHEET 1

Student name:	**School/class**:

1. Fill in the missing sections. Use your own symbols. We have done the first one for you. For the large numbers, write the number instead of drawing the circles. Use a calculator.

Division Story	**Your symbols**	**Change**	**Balance**	**Unknown**
Example: John bought 4 pies. He spent $12. How much is the pie?		Divide 4 by 4		
(a) 4 men scored the same number of points. How much did each man score?				
(b) Mary baked trays of 3 cakes. She baked 24 cakes. How many trays?				
(c) There were $12 meals. Frank spent $108. How many meals altogether?				
(d) They each drove 56km. The total trip was 448 km. How many people drove?				
(e) The 14 girls were paid. The total amount of money was $168. How much did each girl get?				

2. Complete the missing sections with Alan's symbols: as multiply, as divide, as unknown, and as equals. We have done the first for you. Use your own symbols. For large numbers, write the number instead of drawing circles. Use a calculator.

Unknown Story	Symbols	Change	Balance	Unknown
Example: Dad gave all this children $37. It cost him $296. How many children?	◯ ◎ 37 ☰ 296	Divide by 37	◯ ◎ 1 ☰ 8	◯ ☰ 8
(a) The 15 girls all caught the same number of fish. The total number of fish caught was 225. How many did each girl catch?				
(b)	34 ◎ ◯ ☰ 1326			
(c)	◯ ◎ 85 ☰ 5780			
(d) In the relay, everyone cycled 87km. The total distance was 2001km. How far did each person cycle?				
(e)	27 ◎ ◯ ☰ 2187			

MAST MULTIPLICATIVE UNKNOWN: LESSON 3

Objectives:

- To introduce balance principles for <u>division</u> for unknown.
- To use balance principle to solve for unknown in division story.

Materials:

- Counters (preferably natural objects)
- A4 and A5 sheets
- Board set up with:

Counters | blu tack | counters | blu tack | counters

- Calculators, Worksheet 1

Language: Symbol, story, addition, subtraction, multiplication, division, compensation, balance, represent, principal, rule, solving for an unknown

What teacher does:

1. Recap lesson 1 for division with unknown.

   Ask students to construct story: "I shared $12 between 4. Each got $4."

2. State: *We are going to use the balance principle for subtraction as we did for addition in Lesson 2.*

   Do an example. Ask students to construct: "I spent $18 on 3 meals. Each meal cost $6." Ask students to balance the story if the 3 is multiplied by 2.

   Repeat the process for change to first term (the 18). Again ask students to balance the story. Example: "I had $12. I shared it amongst 3 people. Each got $4" – divide 12 by 2.

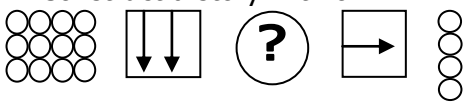
3. Show how to use balance principles with unknowns. Use examples: "I shared the money between 8 people. Each got $7. How much money did I share?"

   Divide the 8 by 2. Discuss how dividing the division number means multiplying the answer (i.e. the less people there are to share, the more each person gets).

   Reinforce the balance rule (What you do to one side of the equals, you do to the other). And show how for division, this means multiplying to reduce the second number.

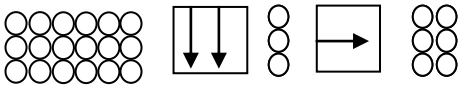
What children do:

Discuss lesson 1. Construct a story with own symbols, e.g.

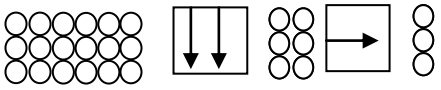


Note: these symbols are examples, students use their own.

Discuss balance principle.
Construct subtraction example with own symbols e.g.

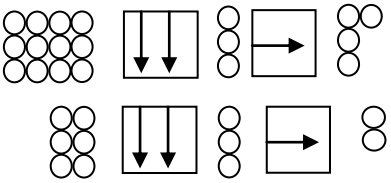


(Multiply 3 by 2 and balance the story)



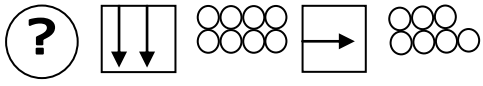
(Multiply 3 by 2 means dividing 6 by 2 to make 3)

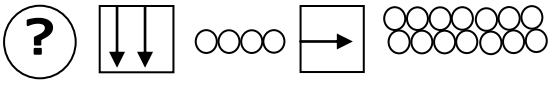
Construct and balance second example:



(Dividing 12 by 2 means dividing 4 by 2)

Construct the story and balance the change with own symbols; e.g.





(Dividing 8 by 2 means multiplying 7 by2)

4. Similar to lesson 2. Show how balance enables unknowns to be calculated. Ask students to construct this division example with an unknown: "amongst 8 and each person got 3"

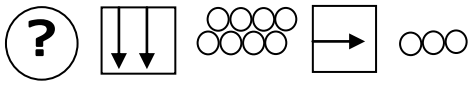
   Ask: *What change to the story will have only the unknown left?*

   Lead discussion to see that if we divide the 8 by 8 to get 1, then the unknown is on its own. This is done by multiplying by 8. Ask students to balance the story with this change.

   Discuss results and show that it gives the unknown value of 24.

   Discuss rule that multiplying a division changes it to 1, therefore, having no effect.

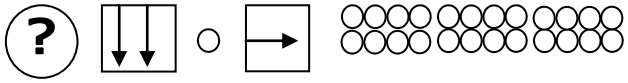
Use own symbols and construct the example:



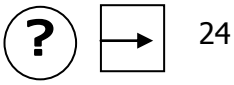
Share symbols and stories. Discuss how could only have the unknown on left-hand-side of equals.

See that dividing by 8 will do this and that this equivalent to multiplying by 8.

Balance when reduce 8 to 1.



Which means:



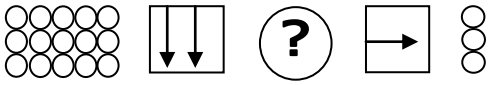
5. Direct students to complete Worksheet 1. Lead discussion of answers. Direct students to write numbers in instead of drawing counters. Give permission to use a calculator. Ensure students know Lyn's symbols.

Complete Worksheet 1. Discuss answers.

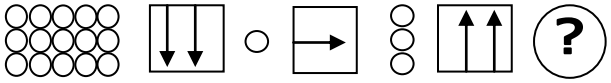
6. Organise students to explore how to use balance to find an unknown when the unknown is the second number e.g. I shared \$15 and each person got \$3. How many people were there?

   Act out with selected students' symbols. (Allow students to use numbers instead of drawings of counters).

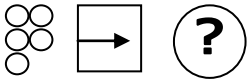
Construct example with own symbols, e.g.



Multiply both sides by 



Divide by 3 to find unknown:



7. Direct students to complete Worksheet 2. Lead discussion of answers.

Complete Worksheet 2. Discuss answers.

Evaluation:

- Students engage and offer opinions.
- Students understand how to use the balance rule.
- Students can complete Worksheets 1 and 2.

MAST MULTIPLICATIVE UNKNOWN 3: WORKSHEET 1

Student name:	**School/class**:

1. Complete the missing sections. We have done the first for you. Use you own symbols. Write numbers instead of drawing circles for counters. Use a calculator.

Unknown Story	Symbols	Change	Balance	Unknown
Example: Sue shared her money between her 7 children. Each child received $4. How much money did she have?	◯ □ 7 □ 4	Multiply 4 by 7	◯ □ 1 □ 28	◯ is 28
(a) Fred spent his money on 36 ice creams. Each one cost $3. How much money did he have?		Multiply 3 by 6		
(b) Jan cut the rope into 17m lengths. She ended up with 15 lengths. How many metres of rope did she have?				
(c) Jack shared the fish amongst 23 families. Each family got 27 fish. How many fish was there to start with?				
(d) In the marathon relay, all riders had to cycle 13km. There were 35 riders. How far did they travel?				

2. Complete the missing sections with Alan's symbols: ⊙ for multiplication, ◡ for division, ◯ for unknown and ☰ for equals. The first is done for you. Write the numbers and use a calculator.

Unknown Story	Symbols	Change	Balance	Unknown
Example: Wendy spent the money on 56 dolls. Each one cost $28. How much money did she spend?	◯ ◡ 56 ☰ 28	Multiply by 56	◯ ◡ 1 ☰ 1568	◯ ☰ 1568
(a) Fred shared the lotto win amongst the 47 members. Each got $38. How much was the win?				
(b)	◯ ◡ 128 ☰ 58			
(c)	◯ ◡ 69 ☰ 156			
(d) John had to cater for 256 guests. They cost $34 each. How much was the catering?				

MAST MULTIPLICATIVE UNKNOWN 3: WORKSHEET 2

Student name:	**School/class**:

1. Complete the missing sections with Alan's symbols: ⊙ for multiplication, ◡ for division, ◯ for unknown and 〓 for equals. The first is done for you. Write the numbers and use a calculator.

Unknown Story	Symbol	Change	Balance	Unknown
Example: I shared $36 amongst the children. Each got $4. How many children?	36 ◡ ◯ 〓 4	Multiply by unknown and divide by 4	Multiply by ◯ 36 ◡ 1 〓 4 ⊙ ◯ Divide by 4 9 〓 1 ⊙ ◯	◯ 〓 9
1. The 42 lollies were shared between the girls. Each got 6 lollies. How many girls?	42 ◡ ◯ 〓 6			
2.	1554 ◡ ◯ 〓 37			
3.	12 054 ◡ ◯ 〓 82			
4. The lotto win of $1274 was shared among 26 people. How much did each person get?				

MAST MULTIPLICATIVE UNKNOWN: LESSON 4

Objectives:	**Materials**:
• To relate invented symbols to formal maths symbols. • To solve for unknowns in formal multiplication and division equations.	• Counters (preferably natural objects) • A4 and A5 sheets • Board set up with: Counters [blu tack] counters [blu tack] counters • Calculators and Worksheets 1 and 2

Language: Symbol, story, addition, subtraction, multiplication, division, compensation, balance, represent, compensation, balance, principal, rule, solving for an unknown

What teacher does:	**What children do:**
1. Recap Lessons 2 and 3. If necessary try an example from the worksheets of the previous lessons.	Discuss what happened in Lessons 2 and 3. Try examples from Lesson 2 and 3 Worksheets.
2. Ask: *What are the formal operations for multiplication, division and equals?* Ask: *Is there a formal symbol for unknown? What is it?* Focus attention on x. Say: *Letters like* x, y, v, a, b *are used as unknowns.*	Discuss the formal symbols: add +, subtract -, multiply x, divide ÷ and equals =. Suggest formal symbols for unknown, e.g: ? or x, y, a, b etc
3. Use formal symbols to represent stories: e.g.: (a) The shop baked 17 trays of bread. This made 595 loaves. How many loaves were on each tray? (b) The shop sold lengths of rope in 32m pieces. Overall the shop sold 1792m of rope. How many pieces were sold? (c) John shared his winnings amongst 9 people. Each got \$8. How much were the winnings? (d) I shared \$3182 lotto winnings amongst the members of the syndicate. Each got \$86. How many people were there?	Use formal symbols to represent stories: (a) $17 \times x = 595$ (b) $x \times 32 = 1792$ (c) $x \div 9 = 8$ (d) $3182 \div x = 86$
4. Ask students to think of the formal symbols as they would for someone else's invented symbols. Say: *Use the methods of Lesson 2 and 3 to solve for unknowns* x. Discuss different students' methods. Remind students that they have to make a change or changes that leave the unknown alone on one side of the equals sign. Then they have to balance the change(s). State that students should show changes in what they write down.	Solve for x: (a) $17 \times x = 595$ (divide by 17) $x = 595 \div 17 = 35$ (b) $x \times 32 = 1792$ (divide by 32) $x = 1792 \div 32 = 56$ (c) $x \div 9 = 8$ (multiply by 9) $x = 8 \times 9 = 72$

(d) $3182 = 86 \times x$

(divide by 86)

$x = 3182 \div 86 = 37$

5. State that it is possible to have more than one operation. Then reverse changes in opposite order.

(a) I bought chocolates and a $4 pie. I spent $10. How much were the chocolates?

Direct students to work out what to do to get unknown on its own. Discuss which should be done first.

Work through 2 or more examples:

(b) I bought burgers at $5. I received a discount of $3. I paid $17. How many burgers did I buy?

(c) I shared the $91 amongst my friends. Each paid $8 for food. This left $5 each. How many friends were there?

Construct symbols for the "chocolate and pie" story:

(a) $3 \times x + 4 = 10$

Work out changes to get x on its own:

(Subtract 4 and divide 3 & balance these changes)

(a) $3 \times x + 4 = 10$

(subtract 4)

$3x = 10 - 4 = 6$

(divide by 3)

$x = 6 \div 3 = 2$

Solve other examples:

(b) $x \times 5 - 3 = 17$

(add 3)

$x \times 5 = 17 + 3 = 20$

(divide by 5)

$x = 20 \div 5 = 4$

(c) $91 \div x - 8 = 5$

(add 8)

$91 \div x = 13$

(multiply by X)

$91 = 13 \times x$

(divide by 13)

$x = 7$

6. Direct students to complete Worksheet 1. Lead discussion of answers. Ask students to show all working.

Complete Worksheet 1. Discuss answers.

Evaluation:

- Students engage and offer opinions.
- Students can differentiate between compensation and balance for both subtraction and addition.
- Students can suggest reasonable rules.

MAST MULTIPLICATIVE UNKNOWN 4: WORKSHEET 1

Student name:	**School/class**:

Complete the missing sections. Use formal symbols. The first two have been done for you.

Unknown Story	Formal Symbols	Change	Working	Unknown
Example: I shared the money with 11 people. They each got $9. How much money did I have?	$x \div 11 = 9$	Multiply by 11	$x \div 11 = 9$ x by 11 $x = 9 \times 11 = 99$	x = $99
Example: At the sale, I bought 6 dresses and pants for $47. I spent $143. How much did I spend on dresses?	$6 \times x + 47 = 96$	Subtract 47 Divide by 6	$6 \times x + 47 = 143$ Subtract 47 $6x = 143 - 47 = 96$ Divide by 6 $x = 96 \div 6 = 16$	x = $16
1. I shared the money with 8 people. They all had $9. They all left with $21. How much did I share?	$x \div 8 + 9 = 21$	Divide 24 by 2		
2. The riders each rode 27km. There was still 54km to go before the 216 race was finished. How many riders?		Divide 12 by 3		
3.	$15 \times x - 6 = 114$	Divide 6 by 3		
4.	$17 \div x + 24 + = 38$	Divide 14 by 7		

Unknown Story	Formal Symbols	Change	Working	Unknown
5.	$x \div 6 = 68 = 75$			
6. All of the bags of lollies were sold. 24 lollies were used in each bag. 480 lollies were used. How many bags were sold?				
7.	$32 \times x + 18 = 2432$			
8.	$2320 \div x - 54 = 26$			
9.	$37 + 47 \times x = 3139$			
10.	$68 + 4462 \div x = 114$			