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Construction and Building

Mathematics behind Dome Construction Using Earthbags

Booklet VC2: Rate, Ratios, Speed and Mixes

DEADLY MATHS VET

Northern Peninsula Area College

'Hands-On learning' Program

Construction and Building

MATHEMATICS BEHIND DOME CONSTRUCTION USING EARTHBAGS

BOOKLET VC2: RATE, RATIOS, SPEED & MIXES

VERSION 1

VC2

Deadly Maths Consortium
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with
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Deadly Maths Consortium
School of Mathematics, Science and Technology Education
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This material has been developed as a part of the Australian School Innovation in Science, Technology and Mathematics Project entitled *Enhancing Mathematics for Indigenous Vocational Education-Training Students*, funded by the Australian Government Department of Education, Employment and Workplace Training as a part of the Boosting Innovation in Science, Technology and Mathematics Teaching (BISTMT) Programme.

YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called “Deadly Maths”.

“YuMi” is a Torres Strait Islander word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology

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08/05/09

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THIS BOOKLET

This booklet VC2 was the second booklet produced as material to support Indigenous students completing courses or certificates associated with construction at the Northern Peninsula Area College, Bamaga. It has been developed for teachers and students as part of the ASISTM Project, *Enhancing Mathematics for Indigenous Vocational Education-Training Students*. The project has been studying better ways to teach mathematics to Indigenous VET students at Tagai College (Thursday Island campus), Tropical North Queensland Institute of TAFE (Thursday Island Campus), Northern Peninsula Area College (Bamaga campus), Barrier Reef Institute of TAFE/Kirwan SHS (Palm Island campus), Shalom Christian College (Townsville), and Wadja Wadja High School (Woorabinda).

At the date of this publication, the Deadly Maths VET books produced are:

VB1: Mathematics behind whole-number place value and operations
Booklet 1: Using bundling sticks, MAB and money

VB2: Mathematics behind whole-number numeration and operations
Booklet 2: Using 99 boards, number lines, arrays, and multiplicative structure

VC1: Mathematics behind dome constructions using Earthbags
Booklet 1: Circles, area, volume and domes

VC2: Mathematics behind dome constructions using Earthbags
Booklet 2: Rate, ratio, speed and mixes

VC3: Mathematics behind construction in Horticulture
Booklet 3: Angle, area, shape and optimisation

VE1: Mathematics behind small engine repair and maintenance
Booklet 1: Number systems, metric and Imperial units, and formulae

VE2: Mathematics behind small engine repair and maintenance
Booklet 2: Rate, ratio, time, fuel, gearing and compression

VE3: Mathematics behind metal fabrication
Booklet 3: Division, angle, shape, formulae and optimisation

VM1: Mathematics behind handling small boats/ships
Booklet 1: Angle, distance, direction and navigation

VM2: Mathematics behind handling small boats/ships
Booklet 2: Rate, ratio, speed, fuel and tides

VM3: Mathematics behind modelling marine environments
Booklet 3: Percentage, coverage and box models

VR1: Mathematics behind handling money
Booklet 1: Whole-number and decimal numeration, operations and computation

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OVERVIEW

The ASISTM VET project funded in 2008 by the Australian Schools Innovations in Science, Technology and Mathematics Scheme had 6 sites: Wadja Wadja High School at Woorabinda, Shalom Christian College at Townsville, Palm Island Post Yr 10 Campus (operated by Kirwan State High School and Barrier Reef Institute of TAFE), Tagai College Secondary Campus at Thursday Island, Thursday Island campus of Tropical North Queensland Institute of TAFE, and Northern Peninsula Area College at Bamaga. Because all these sites have Indigenous students, the project focussed on developing mathematics materials to assist the teaching of the mathematics required for certification for these Indigenous VET students, many of whom have had little previous success in mathematics at school.

Earthbag Mathematics

This booklet was based on mathematics requested by the Northern Peninsula Area College (NPAC). This was with respect to a program run for students in Years 7–10 called "Hands on Learning". Indigenous students at risk of dropping out of school were given one day week learning through a construction and building project if they attended the college the other four days.

The project adopted by NPAC was for the students to use an "Earthbag" construction technique to build circular dome structures for the local council. The first structure was a 6-metre circular dome to act as an information centre. The second structure was a series of domes joined by walkways to act as a display of local history.

Earthbag construction is an inexpensive way to build structures in which cement/lime and earth/sand are mixed to wet, placed in bags which are rammed flat, and then joined together in a circular dome structure. The book *Earthbag Building* by Kaki Hunter and Donald Kiffmeyer describes how to build in this way.

Mathematics areas

As the building was being constructed, NPAC wished to use these opportunities to teach literacy and numeracy. The areas wanted for numeracy or mathematics included:

(1) Circles - properties of circles, including the various components of circles (e.g. diameter, sectors), symmetry, patterns, puzzles, tessellations, dissections etc.).

(2) Circle formulae - including formulae for calculating the circumference and area.

(3) Circle construction – cylinders, cones and spheres, etc., and volumes of these solids.

(4) Circle relationships – perimeter vs area, surface area vs volume, and strength.

(5) Domes – form, construction, and strength.

(6) Building rate, that is, looking at rates at which the construction takes place, using multipliers and multiple line methods for solving problems.

(7) Earthbag mixtures, including meaning of ratio and proportion, and again using multipliers and multiple line methods for solving problems.

Mathematics for Booklet 2

Booklet 2 covers rate. ratio, speed and mixes – the last two points.

Similar to Booklet 1, Booklet 2 is based on the idea that it is important to give students a wide understanding and feeling for rate and ratio as well as how to apply rate and ratio to Earthbag buildings. It looks at multipliers and a technique called double number line to assist students to understand how to solve rate and ratio problems.

Booklet 2 also contains activities and investigations. However, as the sections are moved through, the focus of the booklet tends to remain on straightforward activities with an occasional investigation.

1. MEANING OF RATIO

1.1 Rate vs. ratio

Activity 1.1: Rate and ratio are multiplicative comparisons. Two numbers, for example, 4 and 20, can be compared in each of three ways:

(a) by number: 20 is bigger than 4

(b) by addition: 20 is 16 more than 4

(c) by multiplication: 20 is 5 times as much as 4

Comparison (c) is the meaning behind rate and ratio. If the 20 is money ($) and the 4 is kg of meat, then we can say that the cost of the meat is $5/kg (rate). If the 20 is the number of litres of water and 4 is the number of litres of cordial, then we can say that the water to cordial mix is 20:4 (ratio).

Rate and ratio come from different aspects of our lives and have different histories. Ratio is normally used when attributes are the same, that is, 7 L of water to 2 L cordial gives a ratio of 7:2. Rate is used when there are different measures. That is, $7 is the cost of 2 L of cordial gives us a rate of $7/2 L = $3.50 per 1 L. Thus, in the diagram below, ratio is the shaded areas and rate is the unshaded areas. However, in practice, things are mixed up and sometimes ratio language and symbols are used for rate situations, e.g., mix using 25mL of chemical for each 100m² of wall.

	$	Length	Area	Volume	Mass	Time
$						
Length						
Area						
Volume						
Mass						
Time						

The two different notational forms lend themselves to different interpretations. A ratio such as 2:3 can be related to fractions except that ratio is part to part while fractions are part to whole (note- here the fraction is 2/5). A rate such as $4/L can be seen as a number that multiplies, e.g., $4/L x 3L = $12 (note that the Ls "cancel" and the $ is left).

(1) Compare the sets of two numbers below in each of the three ways mentioned above. Write each as a rate and a ratio:

a. 6 and 4

b. 8 and 24

c. 10 and 15

(2) Determine whether each of the following situations is a rate or a ratio:

a. Sand and cement is mixed mass to mass

b. Sand costs dollars per mass

c. Paint covers area per volume

d. Cordial is mixed with water volume to volume.

(3) State a rate when there are:

a. Area and length

b. Time and money

c. Mass and volume

(4) Construct a ratio when there is:

a. Area to area

b. Mass to mass

c. Temperature to temperature

(5) Determine whether each of the following situations is a rate or a ratio:

a. Butter to milk is 1:2

b. Butter costs $5.40 per kilogram.

c. The paint is applied 2L:10m^2

d. The pressure is 2 kg per cm^2.

e. Number of bottles of coke to bottles of lemonade is 4:3.

(6) Create a relevant rate or ratio problem for:

a. 6:4

b. 4/1

c. 4:7

d. 6.27/1

Investigation 1.1

1. List in your world where you use rates and ratios. Write a list of where your family would use rates and ratios.
2. A foot print is left. It is 28cm long. What height would this person be (calculate ratios of different people's heights to foot length and use this to make a good estimate).

1.2 Models for rate and ratio

Activity 1.2: Rate and ratio can be considered in terms of sets or areas. For example, if cordial is $2 per litre, then the rate can be considered in terms of Diagram A. If cordial is 1:2, then ratio can be considered in terms of Diagram B.

Diagram A:

$2/L can be shown as

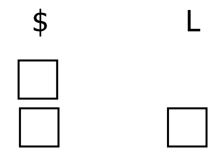
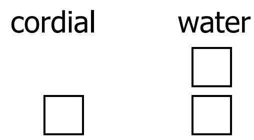


Diagram B:

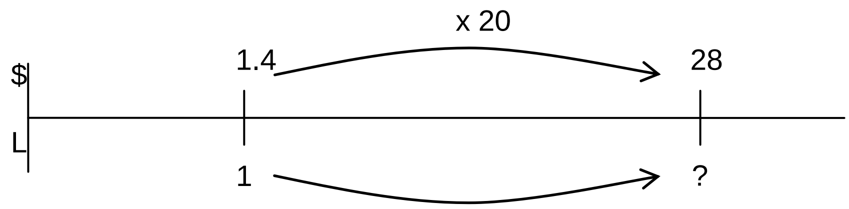
1:2 can be shown as



The consequence of this. First, for both rate and ratio, it enables us to work out answers to problems. For example, if there are 8 L of cordial, then each square is 8 and thus we need $16 to buy the cordial and 16 L of water to mix it up with. Second, for rate and ratio, there is a total as well as the parts. That is, if there are 8 L of cordial and 16 L of water, then this means 24 L of mixture.

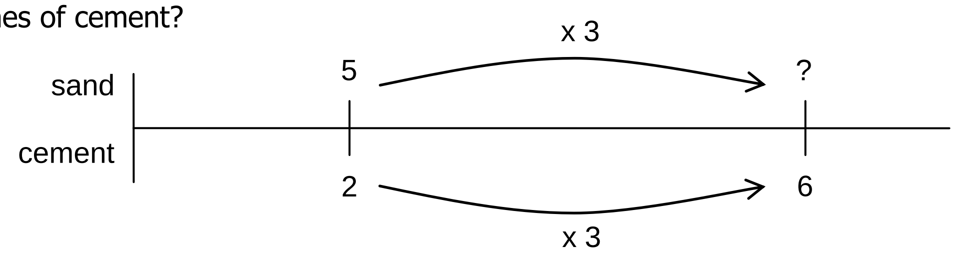
The second model is the double number line. This is a good model for problem solving.

For example, if petrol costs $1.40/L, then how many litres are worth $28?



Thus, the answer is $20.

Another example, sand and cement get mixed in ratio 5:2. How much sand is required for 6 tonnes of cement?



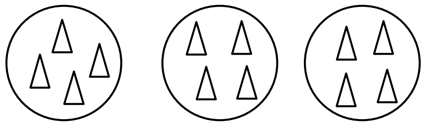
Thus, the answer is 15 tonnes.

It can be seen that the two attributes (for rate) or the two components (for ratio) take a side of the line each and the unknown (?) is worked out, because the change on both sides of the line is the same multiple.

Rates can be directly multiplied as follows:

(1) Three bags of lollies with 4 lollies in each bag

a. Draw a representation:



b. Combine the equal groups: $3 \times 4 = 12$

c. Look at this in terms of rate: 4 lollies/bag × 3 bag = 12 lollies

(2) Repeat this for other examples:

a. 3 trees per row × 5 rows = 15 trees

b. 5 blocks per runner × 6 runners = 30 blocks

(3) Find the pattern:

a. Numbers are modified.

b. Attributes are "cancelled" (for example, lollies ÷ bag × bag = lollies).

Complete each of the following:

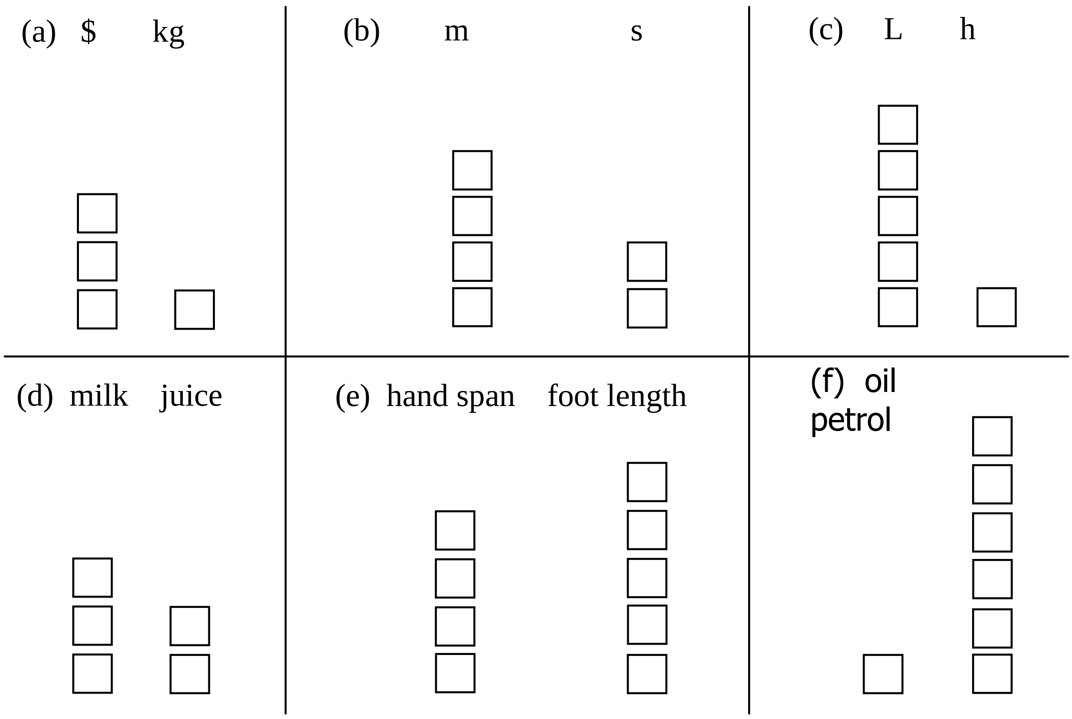
(1) Draw set/area and double numberline diagrams for:

a. Sand to cement is 2:3 (b) Paint is $5/L

b. Baby to man is 1:6 (d) Pressure is 7t/m²

c. Milk to butter is 4:3

(2) State what each of the following rates or ratios are:



(3) Calculate each of the following by set/area models, double number lines and where appropriate (rate multiplication):

a. Cement and sand are mixed 3:5. How much sand for 81t of cement?

b. Petrol is \$1.45/L. How many litres for \$58?

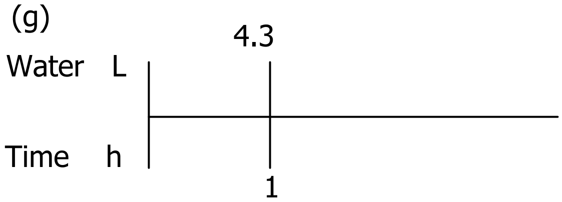
c. Rain fell at 7.5 mm per hour. How much did the swimming pool fill in 4.5 hours?

d. The painter used 0.7 L of paint for 1 m^2. How many litres of paint are needed to cover 7.48 m^2?

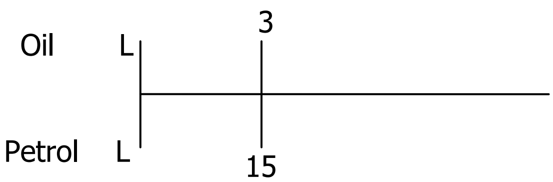
e. A team of dome builders can build a dome in 3 days. How many days will it take the team to build 21 homes?

f. Tie wires must be looped around a strand of barbed wire to hold it in place every 42 cm. How many tie wires must be fitted along a stretch of barbed wire that is 6.2 m long?

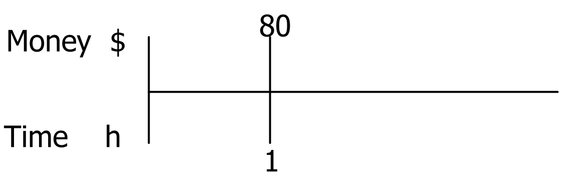
(g)



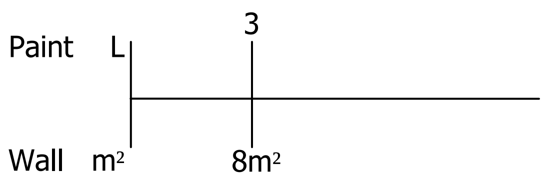
(h)



(i)



(j)



(4) Identify which of these are rate multiplication:

a. 4 apples/box × 7 boxes

b. 6 pencils/box × 3 rulers

c. 5 carriages/train × 6 carriages

8 trucks × 6 cars/truck

Investigation 1.2

There are 2 types of problem for rate and ratio. The first is when you are given the rate or ratio and one component and asked to find one of the components. The second is when you are given both components and asked to find the rate or ratio. Look up how to solve the second type of problem.

Solve these two problems:

(1) Tom bought 21.4L of petrol for \$32.45. What was the price/L?

(2) Jack mixed 64kg of cement with 224kg of sand. What ratio of cement to sand was the mix?

1.3 Stretching and squishing

Activity 1.3: Computers can be used for ratio and proportion, because shapes can be easily changed by pulling on the edges or boundary indicators of the shape.

(1) (a) Use *PowerPoint* to draw two identical people, as shown below:

A B

(b) Change A so that height A:B is 2:1.

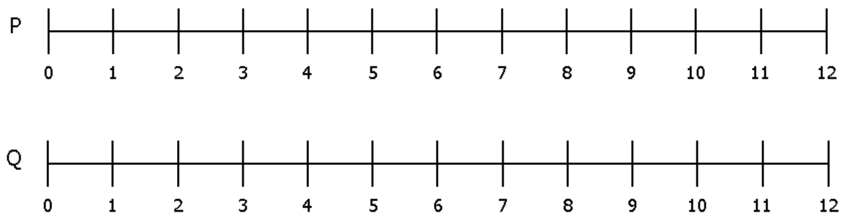
(c) Change B so that width A:B is 2:3.

(d) Change A so that height A:B is 1:3 and width A:B is 1:2.

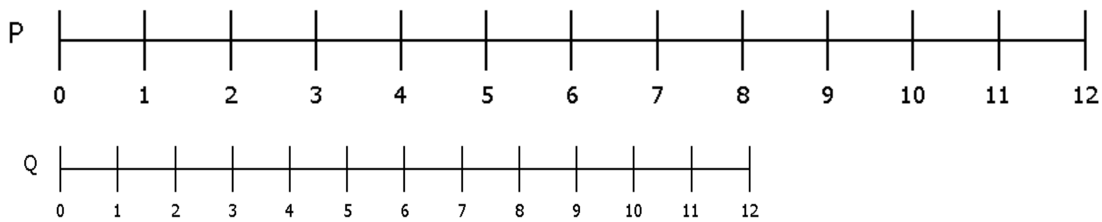
(e) Play with the two people, stretching and squishing them.

(f) Can you make size A:B as 2:1?

(2) (a) Construct two identical rulers P and Q on *PowerPoint*, as shown below:



(b) Stretch ruler P so P:Q is 2:3. That is, the 2 of ruler P aligns with the 3 of ruler Q.



(c) Now, proportions can be seen. For example, if P = 6, then what is Q?

(d) Other ratios can also be seen in the diagram. What other ratios are evident?

2:3 = _____ = _____ = _____ = _____

(3) (a) Construct two identical candles X and Y on *PowerPoint*:

X Y

(b) Change X and Y so that the:

(i) height X:Y is 1:4.

(ii) height X:Y is 3:4.

(iii) width X:Y is 9:1.

(iv) height X:Y is 3:2 and width X:Y is 2:1.

(4) Use your two rulers from Question 2 to show three ratios that are the same as (equivalent to) each of the following:

(a) 1:2 = _____ = _____ = _____

(b) 3:4 = _____ = _____ = _____

(c) 2:5 = _____ = _____ = _____

Investigation 1.3

Construct a drawing on PowerPoint and stretch and squish drawings to calculate the following:

(1) The height of a wall to the width of a buttress is in the ratio 2:1. (a) Calculate the height of a wall when the width of the buttress is 1.5 m. (b) Calculate the width of a buttress when the height of the wall is 3 m.

(2) Earth to soil is in the ratio of 3:7.

a. How much earth is needed when there are 42 kg of soil?
b. How much soil is needed when there are 12 kg of earth?
c. If there are 90 kg of earth and soil in a mixture, then how much earth and soil is in the mixture?

(3) Earth bags to straw bales are in the ratio 5:3.

a. How many earth bags are needed for 27 straw bales?
b. How many straw bales are needed for 55 earth bags?
c. If there are 136 in total, then how many earth bags and straw bales are there?

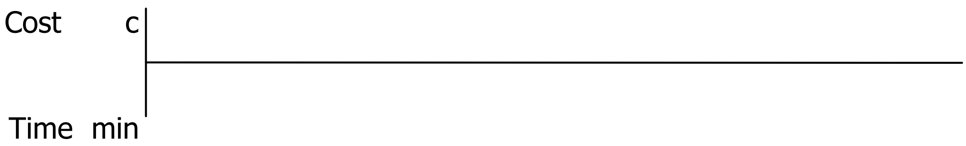
2. USING RATES IN DOME BUILDING CALCULATIONS

2.1 Calculating using rates

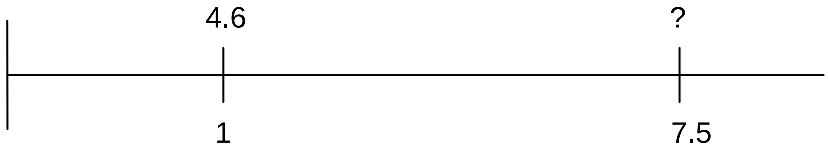
Activity 2.1: If we use a double number line, then rate problems can be easily solved in four steps. Let us look at this example:

A mobile phone call costs 4.6 cents/min, how much does a 7.5 minute call cost?

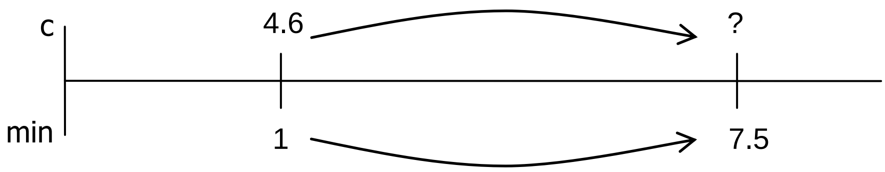
Step 1: Set up the number line (c = cost, m = minutes).



Step 2: Put in what is known. Put ? for the unknown.



Step 3: Work out the multiple. Use the other value to develop a multiplier.



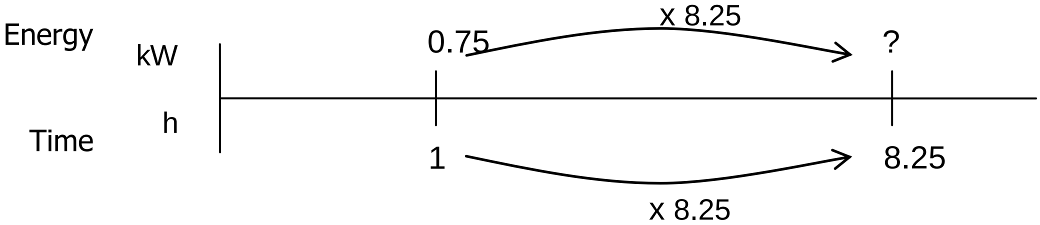
Step 4: Calculate the unknown.

? = 0.46 × 7.5 = $34.5 cents

So, the 7.5-minute call would cost $34.5 cents

Once the double number line is identified, the multiplication of rates is straight forward. Let us look at another example:

The heater uses 0.75 kilowatts per hour (kW/h). If the heater is left on for 8.25 hours, then how many kilowatts were used?

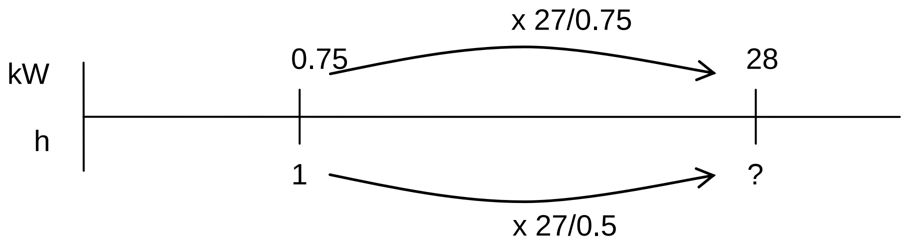


? = 0.75 × 8.25 = 6.1875 kW/h.

So, the heater used 6.1875 kW.

The multiples work in both directions. Let us look at yet another example:

The heater uses 0.75 kW/h. If the heater uses 28 kW, then for how many hours was the heater on?



? = 28 ÷ 0.75 = 21 h.

So, the heater was on for 21 h.

Complete the following for Earthbag buildings

(1) Draw the double number lines for the following:

a. Jenny fills 7 bags/hour

b. Jack rams the bags 11 times/min

(2) Use a double number line to solve each of the following:

a. Jenny fills 7 bags an hour. How many bags in 8 hours?

b. Jack rams bags 71 times/min. How long to ram bags 132 times?

c. Paint for the wall is $12 per litre. How much would 25 litres cost?

d. The mixers motor uses 2.5L petrol/hour. How long can it run on 15L?

e. The bags cost $1.25/bag. How many bags for $100

f. Dome builders are paid $11.50/h. How much will a dome builder be paid for working 6.7 hours?

Investigation 2.1

Investigate whether rates are constant or they change. For example, a small parcel might be $5/kg but a heavier parcel might be reduced to $3/kg or increased to $7/kg. As well, different businesses may charge different rates

(i) Do all rates remain the same? Do some change for different amounts of for different people

(ii) Pick something that is charged as a rate (e.g., mobile phones). Look on internet for different rates – which is the best?

(iii) Some3times rates from different companies are given for different amounts, e.g., sugar is $3.50 for 2kg from company A and $5.20 for 3kg for company B. how do you work out the best buy? (This is particularly difficult for mobile phones)

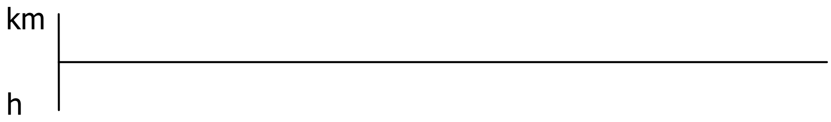
2.2 Calculating rate

Activity 2.2: When a relationship between two amounts is given, there is often a need to calculate the rate. Let us look at this example:

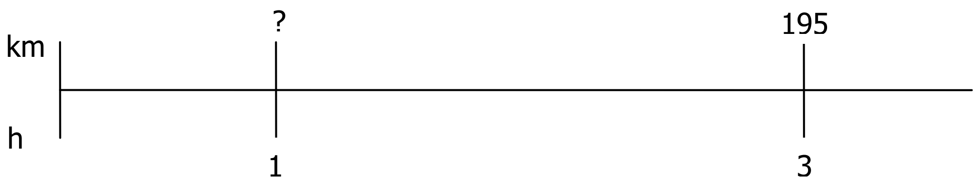
The car travelled 1.95 km in 3 hours. What was the speed?

To do this we need to divide the numbers, or use a multiple that is less than 1.

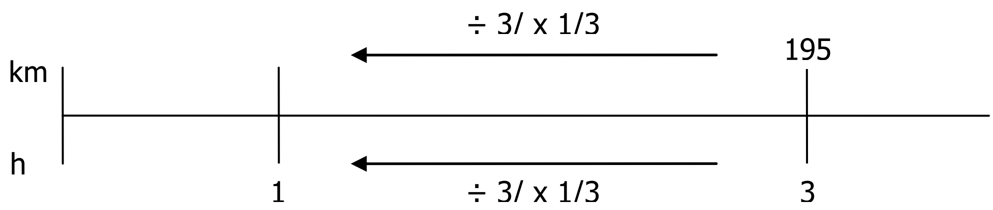
Step 1: In the rate, determine which one is the "per". For example: for $/L the "per" is litres. For km/h, the "per" is hours. The "per" goes on the bottom of the line and the attribute is on the top of the line. In the earlier example, because speed is km/h, hour goes on the bottom and kilometre on the top.



Step 2: Put in the known quantities. Note where the ? is on this example and that the hour is 1 in a rate "per hour".



Step 3: Determine multiplication or division.



Step 4: Calculate the unknown: ? = 195 ÷ 3 = 65 km/h.

Answer each of the following questions using double number lines:

(1) The cost of mobile phone calls was 75c per 30 seconds. What was the rate in terms of cents/s?

(2) A dome builder worked for 11.6 hours and was paid $141.52. What was the dome builder's rate of pay in $/h?

(3) The tap delivered 47 L of water in 5 h. What was the rate in terms of L/h?

(4) The painter covered 20 m² of wall with 8 L of paint. What was the painting rate in terms of m²/L?

(5) It costs $58.75 to have 13.6 tonnes of reject sand delivered. What was the rate in terms of $/t?

(6) Earth bags come in bales of 1000. If one bale costs $140, then what was the rate in terms of $/bag?

(7) A team of dome builders can lay 1320 bags in 7 days. What was the rate in terms of bags/day?

Investigation 2.2

1. Sometimes rates vary for people. For example, if 4 people build a dome house in 5 days, that's a rate of 0.2 domes/day. Now, if there are 8 people building will they double the speed to 0.4 domes/day and build a dome in 2.5 days? Why, why not?

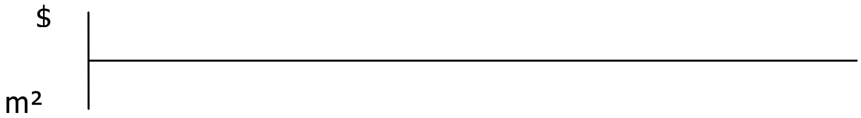
2. Some times other things affect rate. For example, if 1 home handy man can fix the pipes in 2 hours, how long will it take 2 plumbers charging $50/hr? What else can affect rate?

2.3 Applications

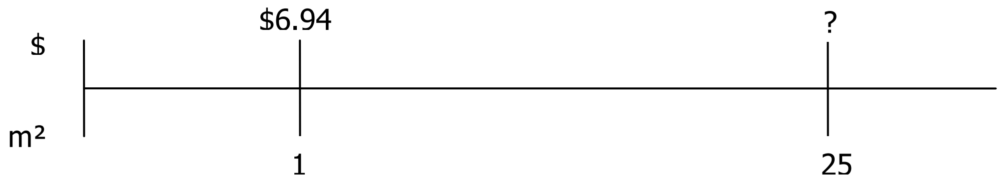
Activity 2.3: Remember that the double number line requires four steps. Have a look each of the following examples:

(1) Basic dirtbag materials cost $6.94 per square metre. How much does it cost to build a dirtbag system that covers 25 m^2?

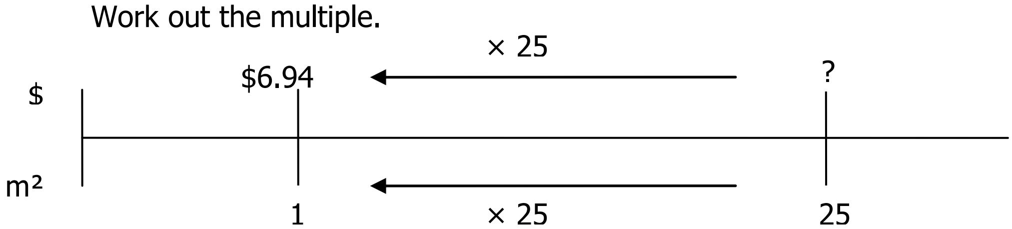
Step 1: Set up the number line.



Step 2: Put in known and unknown quantities.



Step 3: Work out the multiple.



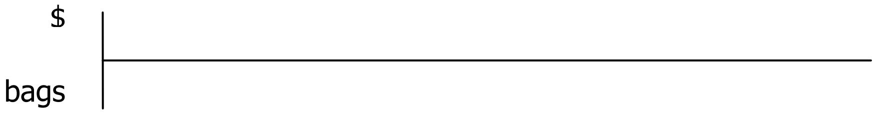
Step 4:

Calculate: ? = $6.94 × 25 = $173.50.

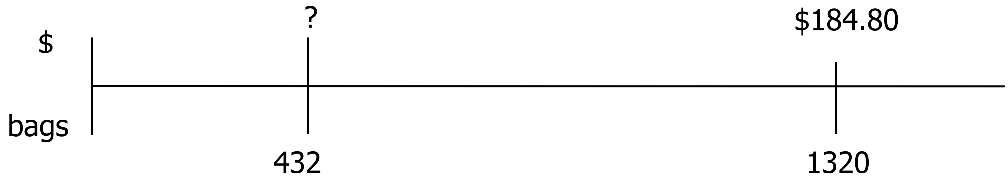
So, the dirtbag materials will cost $173.50.

(2) It cost $184.80 to purchase 1320 bags. How much do 432 bags cost?

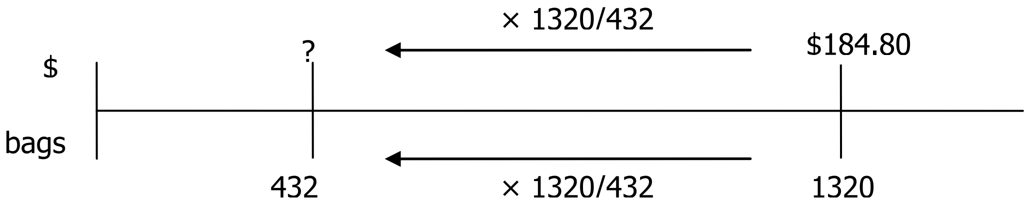
Step 1:



Step 2:



Step 3:



Step 4: ? = 184.80 ÷ (1320/432) = $60.48.

So, 432 bags will cost $60.48.

Complete each of the following:

(1) A boat travels 87 nm in 7 h. What speed was it doing in knots (nm/h)?

(2) A boat travels at 16 nm/h. How many hours will it take to travel 120 nm?

(3) A dome structure with an area of 58 m² requires 47 tonnes of material. How much material will a dome structure with an area of 75 m² require?

(4) A suspended brick setup is required every 60 cm. How many setups are required for a wall with a height of 9 m?

Investigation 2.3

What if there are 3 things, e.g., Earthbag building materials are made at 8.15kg/hours and the builder is paid $37.50 hour. We have $1000. How many kg of material can we make?

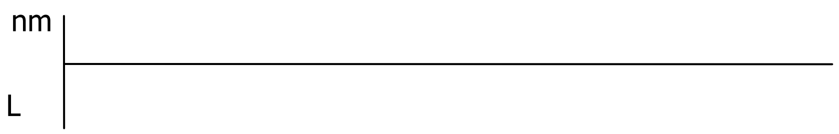
We can use a triple number line.

How can we use this number line?

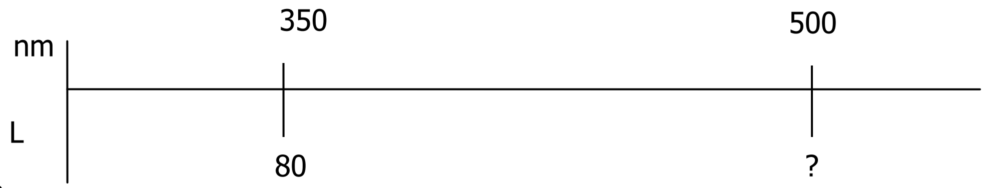
Combination problems can be done without going through the rate. Have a look at each of the following examples.

(1) A boat uses 80 L to travel 350 nm. How much petrol would it need to travel 500 nm?

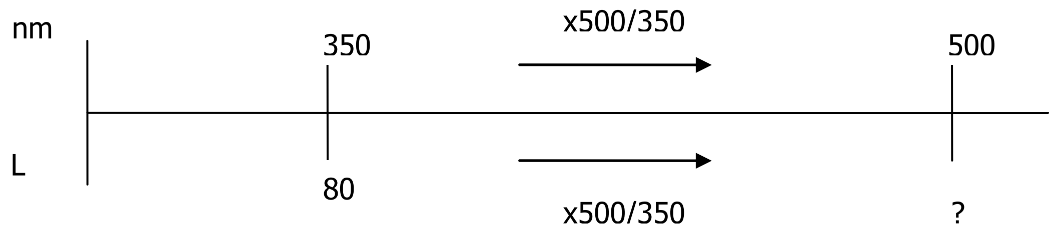
<u>Step 1:</u>



<u>Step 2:</u>



<u>Step 3:</u>



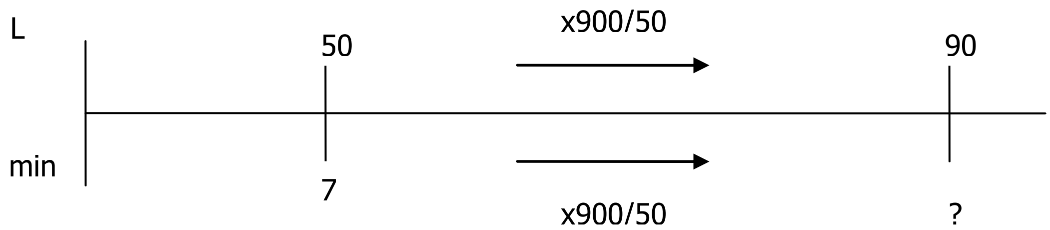
<u>Step 4:</u>

? = 80 × 500 ÷ 350 = 114.3 L.

So, the boat will need 114.3 L of petrol to travel 500 nm.

(2) A refuelling hose delivers 50 L of petrol in 7 minutes. How long will it take to fill a 900 L tank?

<u>Steps 1–3:</u>



<u>Step 4:</u>

Time = 7 × 900 ÷ 50 = 126 mins = 2.1 h

So, the refuelling hose will take 2.1 h to fill the 900 L tank.

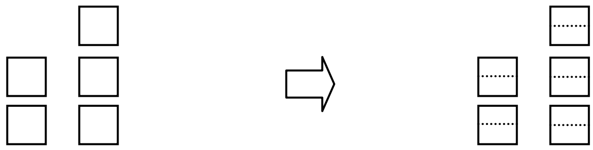
3. USING RATIOS TO MIX MATERIALS

3.1 Understanding proportion

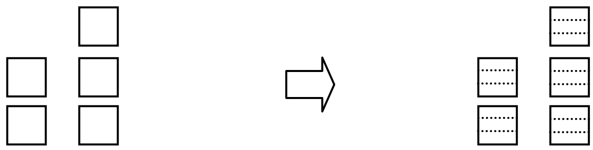
Activity 3.1: There are three ways of dealing with ratio and proportion:

(1) Cutting Squares

Take diagram of 2:3 and cut squares in half. The ratio is now 4:6.



Cut squares into 3 parts, then 2:3 = 6:9.

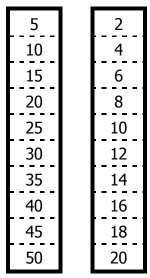


(2) Proportion Sticks

Construct proportion sticks

A ratio 2:5 can be shown by placing the 2 stick beside the 5 stick.

All the equivalent ratios can then be seen: 2:5 = 4:10 = 6:15 = ____



(3) Noticing Multiples

The two ratios 2:3 and 4:6 are in proportion. Notice their multiples. The two ratios 2:3 and 14:21 are also in ratio. Notice their multiples.

$$\overset{\times 2}{\underset{\times 2}{2 : 3 = 4 : 6}} \qquad \overset{\times 7}{\underset{\times 7}{2 : 3 = 14 : 21}}$$

We can now work out proportions by keeping multiples the same.

$$\overset{\times 3}{\underset{\times 3}{3 : 7 = 9 : ?}} \qquad \overset{\times 1/8 \text{ or } \div 8}{\underset{\times 1/8 \text{ or } \div 8}{? : 7 = 24 : 56}}$$

Complete each of the following:

(1) Construct and cut squares to show:

(a) 1:2 = 2:4 = 3:6

(b) 4:1 = 8:2 = 12:3

(c) 3:5 = 6:10 = 9:15

(2) Take ratio sticks as required and complete proportions:

(a) 3:7 = 9:__ = __:28 = 27:__

(b) 6:2 = 36:__ = __:14

(c) 8:5 = 40:__ = __:30

(3) Complete each of the following:

(a)
x2
4:5 = 8:___
x2

(b)
x4
8:5 = ___: 20
x4

(c)
x7
1:4 = ___:28
x7

(d)
x3
___:4 = 9: 12
x3

Investigation 3.1

We have seen that for proportion,

x3
4 : 5 = 12 : 15
x3

We have the same multiples.

Are these multiples the same if they are across each ratio, e.g., are A and B the same?

4 : 5 = 12 : 15

x A x B

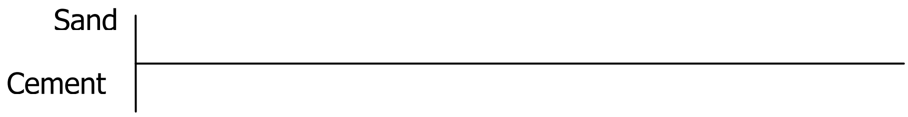
Check this out on a few examples you know are in proportion

3.2 Solving problems using a double number line

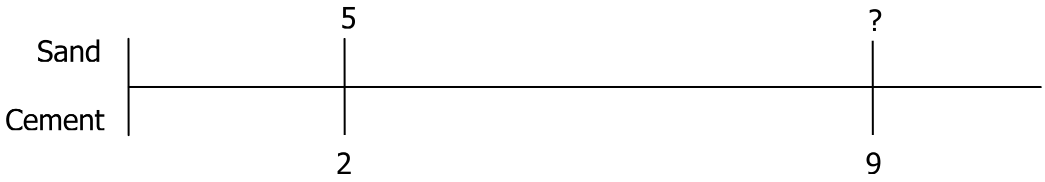
Activity 3.2: Similar to rate, ratio and proportion problems can be easily added with the double number line. Have a look at these examples:

(1) Sand and cement are in ratio 5:2. How much sand is required for 9 tonnes of cement?

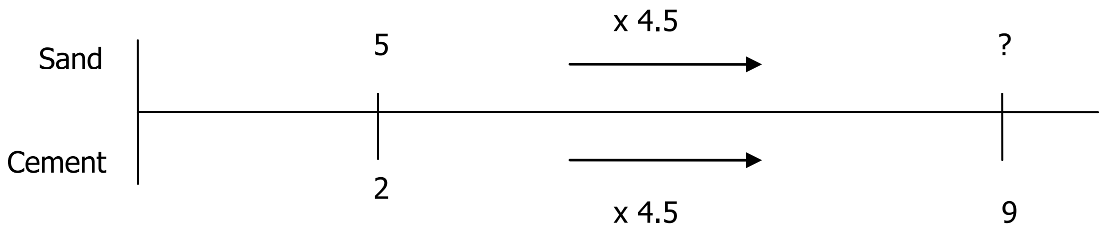
Step 1:



Step 2:



Step 3:



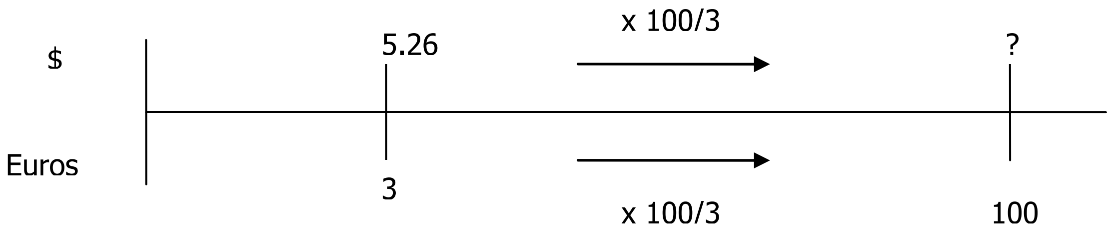
Step 4:

? (Sand) = 5 × 4.5 = 22.5 tonnes.

So, 22.5 tonnes of sand are required with 9 tonnes of cement.

(2) I received 3 euros for $5.26. How many dollars would I get for 100 euros?

Steps 1–3:



Step 4:

? = 5.26 × 100 ÷ 3 = $175.33.

So, 100 euros is equivalent to $175.33.

Complete each of the following activities using a double number line.

(1) Cordial and water is in ratio 2:7. How much cordial is required for 35 litres of water? How much mixture does this make?

(2) Boy to man is 3:5 in height. The boy is 102 cm high. How tall is the man?

(3) Sand to cement is 5:2. How much cement is required for 35 tonnes of sand?

(4) Flour to butter is 4:7. How much flour is required for 50 kg of butter?

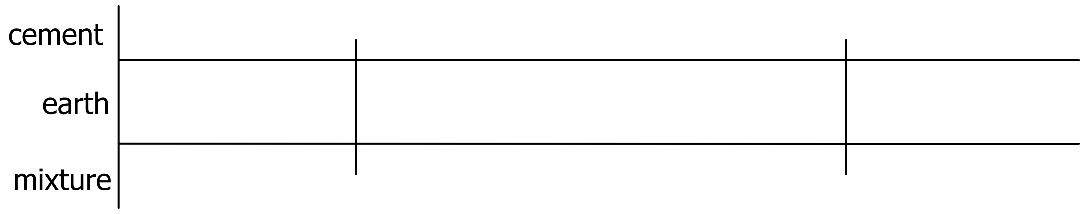
(5) Inch to centimetre is 2:5. How many centimetres are there in 92 inches?

3.3 Mixtures that take into account total amount (the mixture)

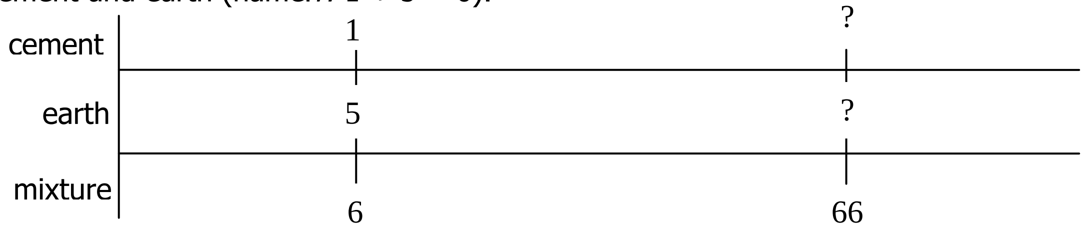
Activity 3.3: A triple number line can be used when solving problems involving mixtures. Let us have a look at this example:

Cement and earth is in the ratio 1:5. How much cement and earth are in 66 kg of mixture?

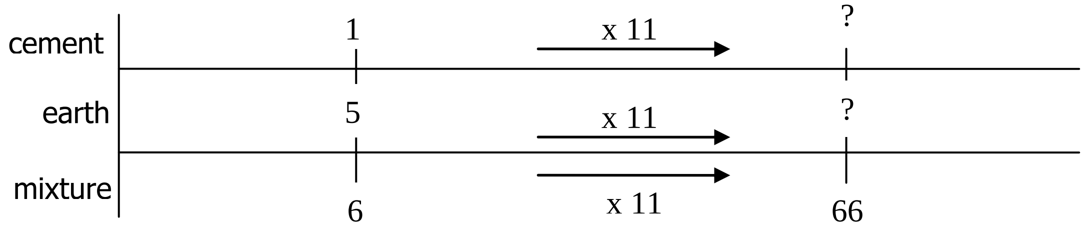
Step 1: Draw the triple number line with the mixture on bottom.



Step 2: Add in the known and unknown quantities. Note that the mixture is the sum of the cement and earth (namely, 1 + 5 = 6).



Step 3: Work out the multipler.



Step 4:

$1 \times 11 = 11$ kg of cement
$5 \times 11 = 55$ kg of earth.

So, in a 66 kg mixture of cement and earth, there are 11 kg of cement and 55 kg of earth.

Investigation 3.3a

1. Can we use ratio like number lines when there are two units? For example, mixing chemicals 25mL with 2L. How much chemical is needed for 7L? Investigate to see if this works – that we do not have to change mL to L or vice versa

2. What about if it is entirely different units, e.g., L to m^2 or $ to km?

Use a triple number line to answer each of the following questions:

(1) Men to women are in the ratio 8:3. How many men and women are in a group of 209 people?

(2) Cats to dogs are in the ratio 5:4. How many cats and dogs are in a shelter of 1269 animals?

(3) Clay to water is in the ratio 3:9. How much clay and water are in 48 kg of a mixture of clay and water?

(4) Earth to soil is in the ratio of 3:7. If there are 90 kg of earth and soil in a mixture, then how much earth and soil is in the mixture?

(5) Earth bags to straw bales are in the ratio 5:3. If there are 136 bags in total, then how many earth bags and straw bales are there?

Investigation 3.3b

Are there examples where things dissolve? E.g., is 1 litre of sugar in 7L of water giving 8L of mixture?

What do we do in these cases?

3.4 Mixtures with three components

Activity 3.4: These need a quadruple number line as follows. They are also what will be used in the domes in Northern Peninsula Area College

Cement, sand, and white ant mounds are mixed in ratio 1:1:5

How much cement for 84kg of mixture? (See

Step 1**:** Draw a quadruple number line

<u>Step 2:</u> Add in the quantities (mixture is sum of 1 + 1 + 5 =)

Cement	1	?
Sand	1	?
White ant	5	?
Mixture	7	84

<u>Step 3:</u> Work out multipliers

Cement	1	$\xrightarrow{\times 12}$	?
Sand	1	$\xrightarrow{\times 12}$	?
White ant	5	$\xrightarrow{\times 12}$	?
Mixture	7	$\xrightarrow{\times 12}$	84

<u>Step 4:</u>

1 x 12 = 12kg cement

1 x 12 = 12kg sand

5 x 12 = 60kg white ant nest

Complete the following

(1) For the ratio cement:sand:white ant of 1:1:5 calculate

a. How much mixture, sand x cement, for 60kg of white ant nest?

b. How much cement, sand and mixture for 50kg of white ant nest?

(2) If you change the mixture to cement:sand:white ant to 2:3:8

a. How much mixture, sand and white ant, for 17kg of cement

b. How much sand, cement, and white ant for 3 tonne (1000kg) of mixture?

Investigation 3.4

Is it possible to have a quintuple number line?

What about if we wanted to calculate taking account of the water?