YUMI DEADLY CENTRE
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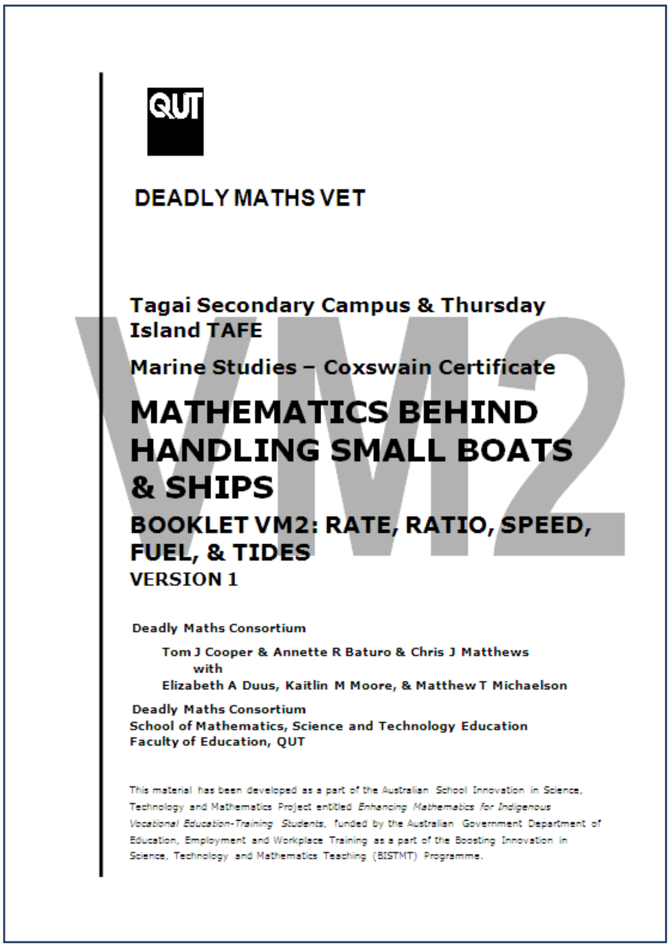
Marine Studies: Coxswain Certificate
Mathematics behind Handling Small Boats and Ships
Booklet VM2: Rate, Ratio, Speed, Fuel, and Tides


DEADLY MATHS VET

Tagai Secondary Campus & Thursday Island TAFE

Marine Studies – Coxswain Certificate

MATHEMATICS BEHIND HANDLING SMALL BOATS & SHIPS

BOOKLET VM2: RATE, RATIO, SPEED, FUEL, & TIDES

VERSION 1

Deadly Maths Consortium

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with
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School of Mathematics, Science and Technology Education
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This material has been developed as a part of the Australian School Innovation in Science, Technology and Mathematics Project entitled *Enhancing Mathematics for Indigenous Vocational Education-Training Students*, funded by the Australian Government Department of Education, Employment and Workplace Training as a part of the Boosting Innovation in Science, Technology and Mathematics Teaching (BISTMT) Programme.

YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology

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BOOKLET VM2

RATE, RATIO, SPEED, FUEL,

AND TIDES

08/05/09

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THIS BOOKLET

This booklet (VM2) was the second booklet produced as material to support Indigenous students completing certificates associated with marine studies/coxswain certificate at the Thursday Island campus of Tagai Secondary College. It may also be applicable to the Coxswain Certificate at Queensland TAFEs, most notably at the Tropical North Queensland TAFE campus at Thursday Island. It has been developed for teachers and students as part of the ASISTM Project, *Enhancing Mathematics for Indigenous Vocational Education-Training Students*. The project has been studying better ways to teach mathematics to Indigenous VET students at Tagai College (Thursday Island campus), Tropical North Queensland Institute of TAFE (Thursday Island Campus), Northern Peninsula Area College (Bamaga campus), Barrier Reef Institute of TAFE/Kirwan SHS (Palm Island campus), Shalom Christian College (Townsville), and Wadja Wadja High School (Woorabinda).

At the date of this publication, the Deadly Maths VET books produced are:

VB1: Mathematics behind whole-number place value and operations
Booklet 1: Using bundling sticks, MAB and money

VB2: Mathematics behind whole-number numeration and operations
Booklet 2: Using 99 boards, number lines, arrays, and multiplicative structure

VC1: Mathematics behind dome constructions using Earthbags
Booklet 1: Circles, area, volume and domes

VC2: Mathematics behind dome constructions using Earthbags
Booklet 2: Rate, ratio, speed and mixes

VC3: Mathematics behind construction in Horticulture
Booklet 3: Angle, area, shape and optimisation

VE1: Mathematics behind small engine repair and maintenance
Booklet 1: Number systems, metric and Imperial units, and formulae

VE2: Mathematics behind small engine repair and maintenance
Booklet 2: Rate, ratio, time, fuel, gearing and compression

VE3: Mathematics behind metal fabrication
Booklet 3: Division, angle, shape, formulae and optimisation

VM1: Mathematics behind handling small boats/ships
Booklet 1: Angle, distance, direction and navigation

VM2: Mathematics behind handling small boats/ships
Booklet 2: Rate, ratio, speed, fuel and tides

VM3: Mathematics behind modelling marine environments
Booklet 3: Percentage, coverage and box models

VR1: Mathematics behind handling money
Booklet 1: Whole-number and decimal numeration, operations and computation

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OVERVIEW

Preamble

The Coxswain Certificate is an important vocational course for Indigenous students who live on the coast or on Islands (e.g. Palm Island, Torres Strait Islands). In their situations boats are widely used and often are the main form of transportation. The Coxswain Certificate is necessary if the boats are large or used for commercial activity.

Mathematically, there are two particular difficulties in the Coxswain Certificate:

(1) Navigation: This is based on position, direction and distance that are both based on angle (in degrees, minutes, and 10ths of minutes). It requires understanding compass bearings in practice on boats and in planning on maps. It also requires changing the course to take into account magenetic variation of the location and further changing this to take account of boat variations.

(2) Travel and Tides: Rate and ratio are important in planning travel. The amount of fuel needed is determined by speed (knots which are nautical miles/hour) and consumption (litres per hour), both of which are rates. The time at which a boat can enter and leave a mooring is determined by relationships between tides, time and draughts of boats, all of which are based on ratio/proportion.

Thus, this book VM2 is based on travel and tides with previous book, VM1, being on navigation. The focus of this booklet is on the following mathematics:

a. introducing rate and ratio - how they are a way of using multiplication for comparing;

b. showing how rate & ratio problems may be solved by using multiples and number lines;

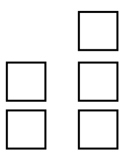
c. applying rate to problems & distance, time and speed and to fuel use; and

d. applying ratio to tides and times to enter and leave ports.

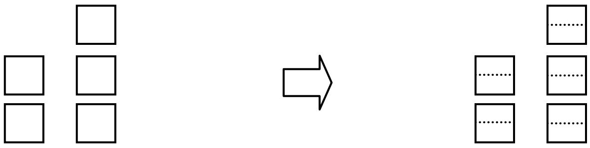
Mathematics behind ratio

(1) Ratio is a multiplicative way of identifying comparative relationships similar to fractions but relating part to part instead of part to whole. Ratio is normally for like attributes, e.g., length to length, mass to mass or time to time. Multiplicative comparison across different attributes is usually called rate. However, in practice things can become confused and ratio has been used for different attributes, e.g., 2L:7m^2 for 2L paint covers 7 square metres.

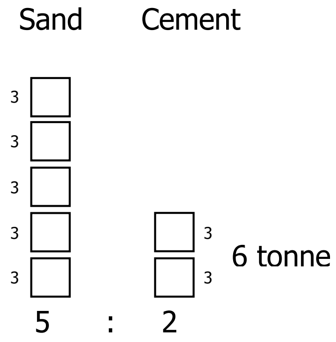
(2) Set/*Area Model*- A good model for understanding ratio is based on sets/areas, e.g., 2:3 as in the figure below. One part is 2 and the other part is 3 with the total being 5.



Ratios can be equivalent, i.e. 2:3 can become 4:6 as shown in the figure below (where each square is divided in two). Equivalent ratio is called proportion.

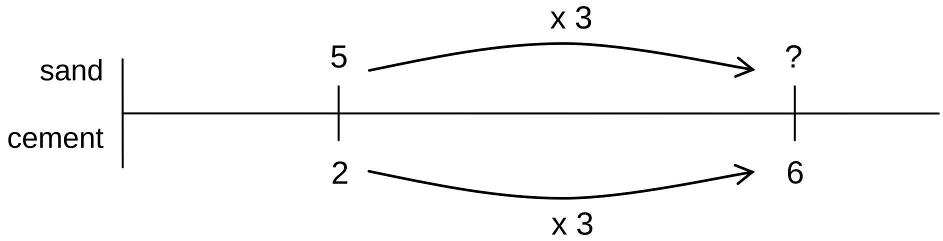


Proportion problems can be solved with the set/area model, e.g., sand:cement is 5:2. How much sand for 6 tonne of cement?



6 tonne of cement is 2 squares, 3 tonne a square, making sand 15 tonne.

(3) *Double Number Line*- Problems can also be solved using the double number line (the change on both top & bottom is the same multiple)



(4) *Proportion Stick*-

Difficulties with equivalent ratios or proportion can be helped by proportion sticks as on right.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

5	2
10	4
15	6
20	8
25	10
30	12
35	14
40	16
45	18
50	20

For 5:2, put the 5 stick beside the 2 stick (as on right); this shows that 5:2 = 10:4 = 15:6 and so on.

Mathematics behind rate

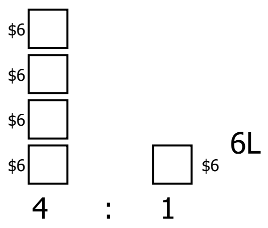
(1) Rate and ratio are different in 3 ways. Ratio compares the same attributes (for example, sand and cement in mass are 5:2 – that is 5 tonnes of sand is used with 2 tonnes of cement); rate compares different attributes (for example, the cost of petrol

is $1.20 per litre which compares money to volume). Note: In practice rate and ratio get mixed up – 2L paint covers $7m^2$ of wall can be considered as $2L:7m^2$ and 11mL of chemical to 2L of water can be considered as 5.5mL/L.

Ratio uses notation similar to common fractions in that there are 2 whole numbers (but part to part, not part to whole (e.g. sand to cement is 5:2); while rate usually uses a single number (e.g. the price of petrol is $1.20 per litre). Thus it is related to decimal fractions but it uses a single number. However, rate could be considered as a ratio with 1 as the second number (for example, petrol's price could be considered as $s to litres is $1.20:1L).

Ratio problems are worker out by using proportion or equivalent ratio (for example, if sand to cement is 5:2 and we wish to make 8 tonnes of cement, then we need 20 tonnes of sand as 20:8 is the same as 5:2); rate problems use multiplication (for examples, if petrol is $1.20 per litre then 4 litres is $1.20 x 4 = $4.80). This is because multiplication is really rate by number (for example, 3 lollies/bag x 4 bags = 12 lollies). Note that the attribute for the number is the same as the second attribute for the rate and the answer is the first attribute to the rate.

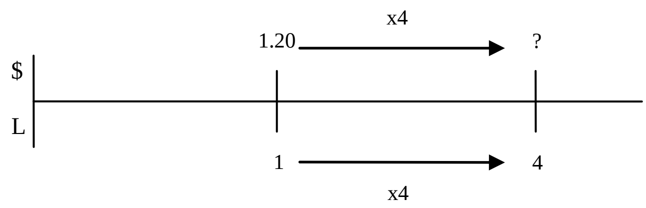
(2) Similar to ratio, rate problems can be considered as set/areas for example, if liquid is $4/L. How many litres for $24?



$24 is 4 squares, so one square is $6.

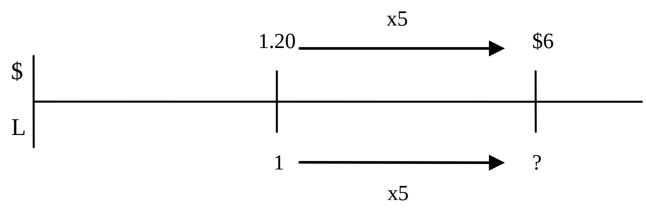
Thus amount of litres is 6

(3) Another way is the double number line. In this method, the top of the line is one attribute while the bottom of the line in the other. So $1.20/litre and 4 litres is:



The answer is $1.20 x 4 = $4.80

This works both ways: e.g., $1.20/litre and have $6.00 =



Answer is 5 litres

(4) There are 3 Problem Types:

a) given a rate, find the first attribute for an amount of the second attribute (e.g., the rate is \$2/L and there is 7L, then the cost is \$14 – this uses multiplication);

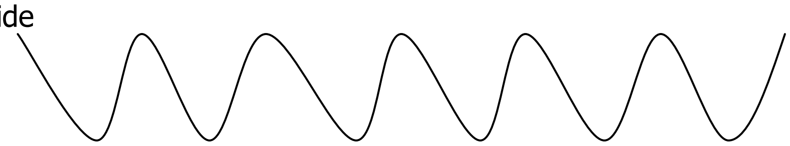
b) given a rate, find the second attribute for an amount of the first attribute (e.g., the rate is \$2/L and there is \$10, then the volume is 5L); and

c) given both attributes, find the rate (e.g., attributes are \$12 and 4L, thus the rate is 12÷4 = \$3/L).

Rates are calculated by ensuring that the second attribute is 1. So, \$8 for 2L is 8 ÷ 2 or \$4/L. Similarly, 34 tonnes in 5 hours is 34÷5 = 6.8 tonnes/hour.

Mathematics behind tides

(1) Tide fluctuates under the influence of the moon and follows a sine curve with the length of tides from low to high or high to low being approximately 6 hours.



If at a harbour, depth of tides varies from high of, say, 3.3m, to a low of, say, 0.9m. If a boat draws 2m, then the boat can only come in near high tide. As high tide and low tide times are known, this enables times to be worked out when the boat can enter the harbour without running aground.

(2) Working out the times a boat can enter a harbour required relating high and low tide depth and time differences (this requires subtraction to work out differences):

Tide	Depth (m)	Time (h)
Low	0.9	2.30am
High	3.3	8.30am
Difference	2.4m	6h

(3) Finding the time when the depth is first OK means relating it proportionally to the time. Adding, we get 0.9m + 1.1m = 2m. Therefore, the 2m depth boat can enter the harbour when the tide is 1.1m above low tide or 11/24ths or 46% along the depth difference from low to high (a little less than half way). This would normally mean 46% of the way along the time difference of 6 hours (all tides are approximately 6 hrs). Now 46 % of 6 hours is 2.7 hours or 2 hours 46 minutes. This would mean that the 2m depth boat could enter the harbour at 2.30 + 2.46 = 5.16am. However, a 2m boat entering at this time was run aground because tides are not linear.

(4) Because the tide is a sine curve, the tide depth is not linear – it changes slowly at the start, accelerates upwards at a fast rate halfway through the tide change, and then slows down at the end. Without a scientific calculator or a computer, the rule of

thumb for this change is 1/12 for 1^{st} hour, 1/6 for the 2^{nd} hour, 1/4 for the 3^{rd} hour, 1/4 for the 4^{th} hour, 1/6 for the 5^{th} hour and 1/12 for the last hour. This means that, in tides, time and distance do not relate linearly as in (P) but non-linear as in (Q).

P

Depth	Time
High	High
5/6	5/6 (5h)
4/3	4/3 (4h)
1/2	½ (3h)
2/3	2/3 (2h)
1/6	1/6 (1h)
Low	Low

Q

Depth	Time
High	High
11/12	5/6 (5h)
3/4	4/3 (4h)
1/2	½ (3h)
1/4	2/3 (2h)
1/12	1/6 (1h)
Low	Low

If we put in the depths and times, this gives the following height-time relationship. (NOTE: for tide differences in 2.4m, each 1/12 is 0.2m, so the first and last hour are 0.2m, the 2^{nd} and 5^{th} hour are 0.4m and the 3^{rd} and 4^{th} hour are 0.6m).

Depth	Time
3.3m	8.30am
3.1m	7.30am
2.7m	6.30am
2.1m	5.30am
1.5m	4.30am
1.1m	3.3am
0.9m	2.30am

Now it is easy to see that a 2m boat should wait to just before 5.30 before entering.

By reversing the direction of the tide (high to low), we have the tide pattern for the next 6 hours:

Depth	Time
0.9m	2.30pm
1.1m	1.30pm
1.5m	12.30pm
2.1m	11.30am
2.7m	10.30am
3.1m	9.30am
3.3m	8.30am

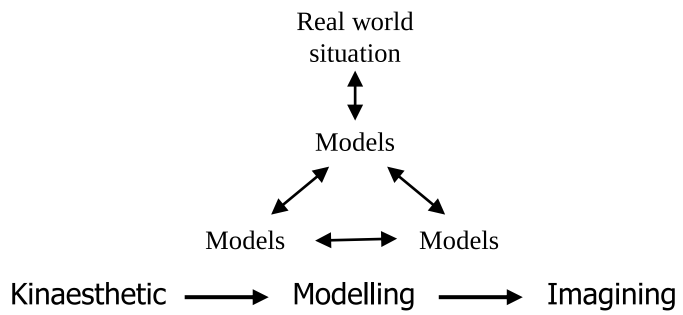
We can see that the boat would have to leave by just after 11.30am or it will be aground.

Pedagogy

(1) Place all mathematics teaching possible within the context of coxswain activity and within the context of the students (Island living and Torres Strait Islander culture).

(2) Focus mathematics teaching on mathematics as a whole structure, its pattern and interactions, using the learning style advocated by Uncle Ernie Grant. This means that mathematics teaching needs to focus on understanding how mathematics is related, sequenced and integrated, that is, on the connections between mathematics ideas.

(3) Where possible begin all teaching using whole body of students (kinaesthetic) then move to using materials, computers and pictures (modelling) and then to having the student think of a "picture in the mind" (imagining), that is, use a variety of representations that follow this sequence:



(4) Relate real world situations (within the context of the students) to models of the mathematics and then to language and finally to symbols (the Payne-Rathmell triangle as on right)

(5) Use the generic pedagogies of generalising, reversing and being flexible, that is, ensuring that the most general understanding possible is developed, that activities move in both directions (model to symbol and symbol to model), and that students have flexible understandings of concepts and processes (including language).

1. NATURE OF RATE AND RATIO

1.1 Rate vs. Ratio

(1) Rate and ratio are multiplicative comparison. Two numbers, 4 and 20, can be compared in 3 ways:

compared in 3 ways:	a) by number-	20 is bigger than 4
	b) by addition-	20 is 16 more than 4
	c) by multiplication-	20 is 5 times as much as 4

Comparison (c) is the meaning behind rate and ratio. If the 20 is money ($) and the 4 is Kg of meat, we can say that the cost of the meat is $5/Kg (rate). If the 20 is the number of litres of water and 4 is the number of litres of cordial, we can say that the water to cordial mix is 20:4 (ratio).

Note: in the coxswain course we use rate for speed (knots/hour) and fuel (litres/hour) and ratio for tides and times.

(2) Rate and ratio come from different aspects of our lives (have different histories). Ratio is used when attributes are the same, that is, 7L of cordial to 2L water gives a ratio of 2:7. In this, it is similar to fraction in showing two numbers, but ratio is part to part while fraction in part to whole (e.g., the fraction of cordial for ratio cordial:water of 2:7 is 2/9 because the whole is 2+7=9).

Rate is used when there are different measures, that is, $7 is the cost of 2L of cordial. It is calculated by division as a decimal, that is, when 2L costs $7, rate is 7 divided by 2 or $3.50/L.

Thus, in the diagram below, ratio is normally the shaded areas and rate is normally the unshaded.

	$	Length	Area	Volume	Mass	Time
$						
Length						
Area						
Volume						
Mass						
Time						

In practice, things can get mixed up and sometimes ratio is used for different units (e.g., considering paint being applied 2L for each $7m^2$ as ratio $2L:7m^2$). Similarly, rates could be considered as ratios where the second number is 1 (e.g., rate of $1.25/L could be considered as ratio $1.25:1L).

(3) Complete the following:

(i) Compare the sets of two numbers below in the 3 ways. Write each as a rate and a ratio:

a) 6 and 4
b) 8 and 24
c) 6 and 4

(ii) Mark the following as a rate or ratio:

a) Sand and cement is mixed mass to mass
b) Sand costs dollars per mass
c) Paint covers are per volume
d) Cordial is mixed with water volume to volume

(iii) State a rate for where there is:

a) Area and length
b) Time and money
c) Mass and Volume

(iv) Construct a ratio where there is:

a) Area to area
b) Mass to mass
c) Temp to temp

(v) Mark the following as rate or ratio or both:

a) Butter to milk is 1:2
b) Butter costs $5.40 per kilogram
c) The pressure is 2Kg per square cm
d) No. of bottles coke to bottles of lemonade is 4:3
e) Paint to area is 2L:7m^2

(vi) Make up a relevant rate or ratio problem for:

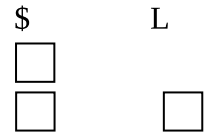
a) 6:4
b) 4.7/1

1.2 Models for rate and ratio

(1) Rate and ratio can be considered in terms of **sets/area**. For example, if cordial is $2 per litre, the rate can be considered in terms of diagram A. If cordial is 1:3 then ratio can be considered in terms of diagram B.

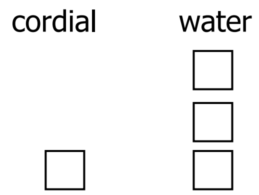
A

$2/L can be shown as



B

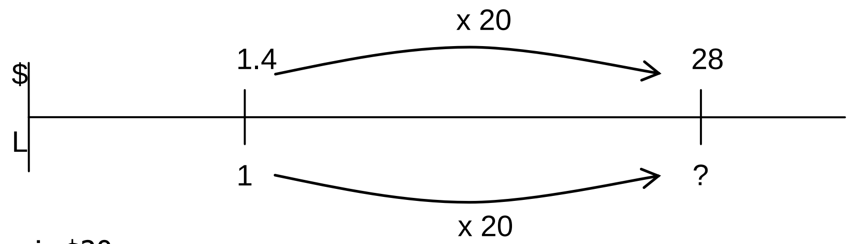
1:3 can be shown as



These diagrams enable us to work out answers to rate and ratio problems. For example, if there is 8L of cordial, it means that each square in A is 8 and thus we need \$16 to buy the 8L of cordial. Similarly, in B, 8L of cordial makes each square 8 and thus we need 24L of water to mix the cordial with. Second for rate and ratio, there is a total as well as the parts – if there are 8 L of cordial and 16 L of water, then this means 24L of mixture.

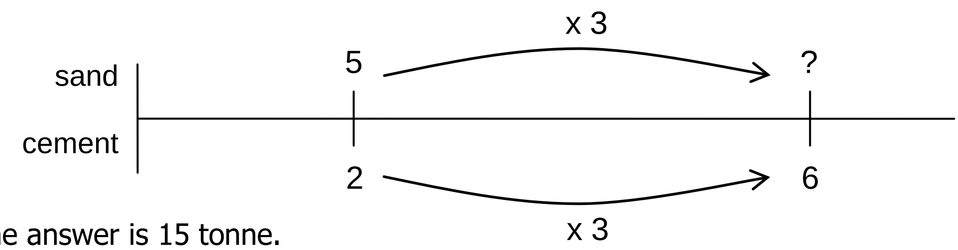
(2) The second model is the **double number line**. Rates and ratios are put on either side (top and bottom) of the line and numbers are placed. Answers are determined from multipliers being the same for top and bottom. This is a good model for problem solving as can be seen in the examples below. (See section 3 & 4 for more on this.)

For rate example, "Petrol is \$1.40/L, how many litres for \$28?", the double number line acts as follows.



Thus the answer is \$20.

For ratio example, "sand and cement get mixed in ratio 5:2, how much sand for 6 tonnes of cement?", the double number line acts as follows.



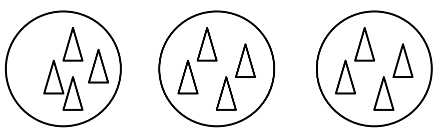
Thus the answer is 15 tonne.

As described above, the two attributes (for rate) or the two components (for ratio) take a side of the line each and the unknown (?) is worked out because the multiplier on both sides of the line is the same multiple.

(3) Rates can be **directly multiplied** as follows:

Three bags of lollies with 4 lollies in each bag

a. Draw a representation:



b. Combine equal groups, that is, 3 x 4 = 12

c. Look at this in terms of rate, that is, 4 lollies/bag x 3 bag = 12 lollies

Repeat this for other examples

a. 3 trees per row x 5 rows = 15 trees

b. 5 blocks per runner x 6 runners = 30 blocks

Find that the pattern shows the following:

a. Numbers are modified

b. Attributes are "cancelled" (for example, lollies ÷ bag x bag = lollies)

(4) Complete the following

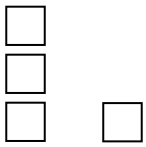
(Note: The double number line will be applied in sections 3 & 4)

(i) Draw diagrams for

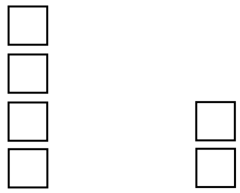
a) Sand to cement is 2:3
b) Baby to man is 1:6
c) Milk to butter is 4:3
d) Chemical is $5/L
e) Pressure is 3t/m^2

(ii) State what these rates/ratios are

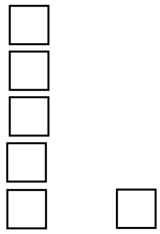
(a) $ kg



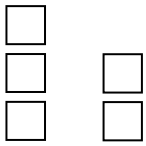
(b) m sec



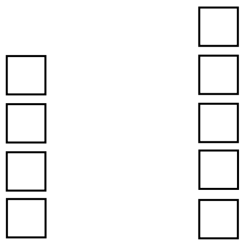
(c) L Hr



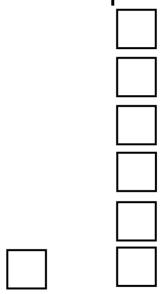
(d) milk juice



(e) hand span foot



(f) oil petrol



(iii) Identify which of these are rate multiplication

a) 4 apples/box x 7 boxes
b) 6 pencils/box x 3 rulers
c) 5 carriages/train x 6 carriages
d) 8 trucks x 6 cars/truck

(iv) Calculate the following by rate multiplication (use calculator)

a) Rain fell at 7.5mm per hour. How much did the swimming pool fill in 4.5 hours?

b) The painter used 0.7L of paint for 1m². How many litres of paint is needed to cover 7.48m².

1.3 Stretching and squishing

(1) Computers can be used for ratio and proportion because shapes can be easily changed by pulling on the edges or boundary indicators of the shape

Use PowerPoint to draw 2 identical people

A B

(i) Change A so that height A:B is 2:1

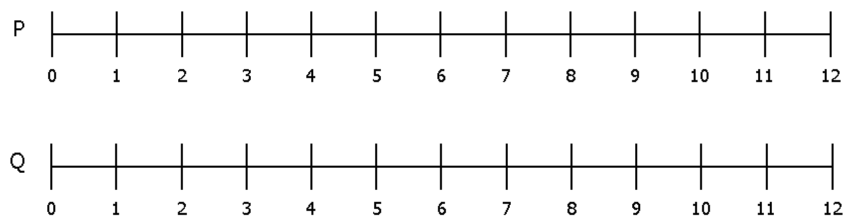
(ii) Change B so that width A:B is 2:3

(iii) Change A so height A:B is 1:3 and width A:B is 1:2

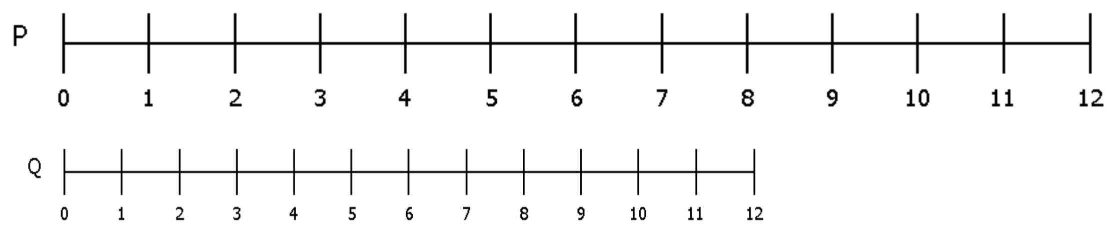
(iv) Play with the two people, stretching and squishing.

(v) Can you make size A:B as 2:1?

(2) Construct 2 identical rulers P and Q on PowerPoint



Stretch P so P:Q is **2:3** – that is, the 2 of P aligns with the 3 of Q.



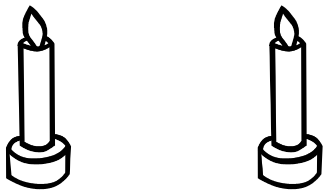
Then, proportions can be seen, for example – "If P = 6, what is Q?". [Answer 9 as is evident from the diagrams.]

Other ratios can also be seen in the diagram – "What other ratios are evident?".

2:3 = _____ = _____ = _____ = _____ ?

(3) Complete the following:

(i) Construct 2 identical candles X and Y on PowerPoint



X Y

Change X and Y so that

a) height X:Y is 1:4

b) height X:Y is 3:4

c) width X:Y is 9:1

d) height X:Y is 3:2 and width X:Y is 2:1

(ii) Use your two rulers to show three ratios the same as (equivalent to)

a) 1:2 = _____=_____=_____

b) 3:4 = _____=_____=_____

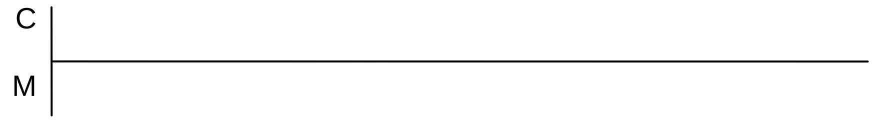
c) 2:5 = _____=_____=_____

2. USING RATE TO CALCULATE SPEED AND FUEL CONSUMPTION

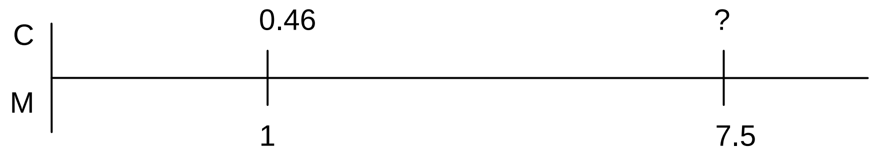
2.1 Calculating using rates

(1) If we use a double number line, rate problems can be easily solved in 3 steps. Let's look at example "a mobile phone call costs 0.46 cents/min, how much for a 7.5 minute call. Use 4 steps

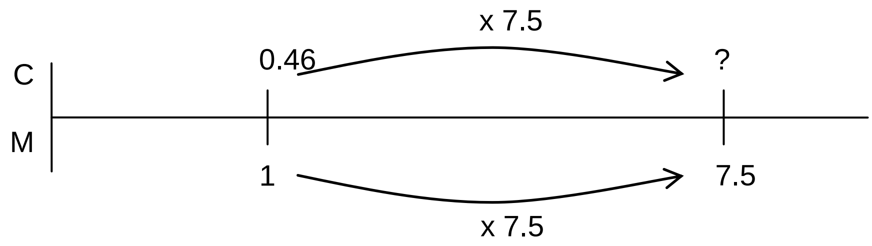
Step 1 Set up number line



Step 2 Put in what has to be found. Put ? for unknown



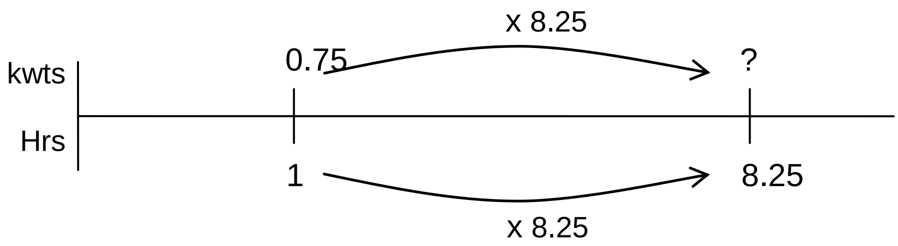
Step 3 Work out the multiplier. Use the other values to determine the multiple.



Step 4 Calculate the unknown

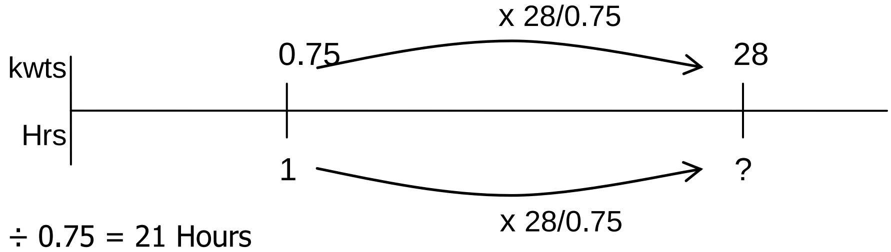
? = 46 x 7.5 = 345c or $3.45

(2) Once the double number line is set up and the multiplier/multiple is identified, the multiplication of rates is straight forward. For example: "The heater used 0.75 kwts/hour. If the heater is left on for 8.25 hours, how many kilowatts were used?".



? = 0.75 x 8.25 = 6.1875KW/H

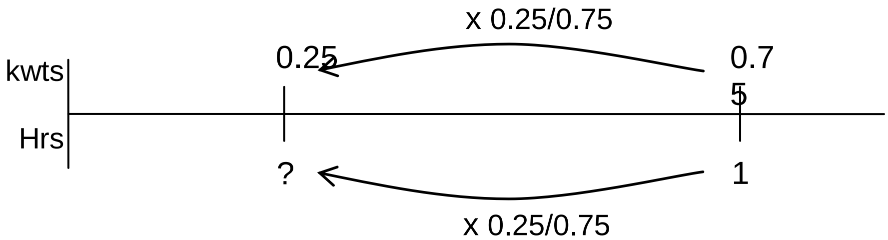
(3) The multiples work on both sides of the line. For example: "The heater uses 0.75 kwts/hour. If the heater uses 28 kwts how many hours was the heater on?"



? = 28 ÷ 0.75 = 21 Hours

(Note: This example uses the simplest way to work out multiples/multipliers in complex situations. It simply divides the second number by the first – e.g., 1st number is 0.75, 2nd number is 28, so multiplier is 28/0.75.)

(4) The multiples work in both directions. For example: "The heater uses 0.75 kwts/hour. If the heater uses 0.25 kwts, what part of an hour was the heater on?"



? = 1 x 0.25 ÷ 0.75 = 1/3 hour or 20 minutes.

(5) Complete the following:

(i) Draw the double number lines for the following:

a) Speed of boat is 13 knots (13 n.m/hr)

b) Use of fuel is 1.5 L/n.m.

(ii) Use a double number line to solve the following:

a) Paint is $12 per litre. How much for 25 litres?

b) Smarties were $17.06/kg. How much for 1.35kg of smarties?

c) Paint is $12 per litre. How many litres for $72?

d) Smarties were $17.60/kg. How many kg's of smarties for $50?

e) Cleaning is $16/hour. How much does it cost for 7.25 hours of cleaning?

f) The water pump delivers 235L/hour. How many litres for 15 hours of pumping?

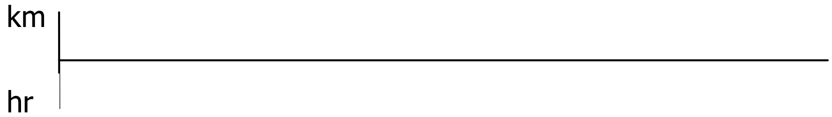
g) The water pump delivers 235L/hr. How many hours to deliver 1000L?

2.2 Calculating rate

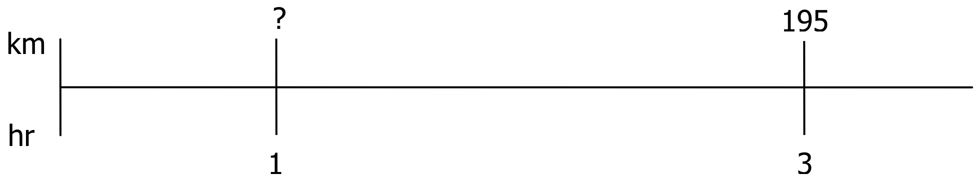
(1) When a relationship between two amounts is given there is often a need to calculate the rate. For example: "The car travelled 195 km in 3 hours. What was the speed?"

To do this we need to divide the numbers (e.g., 195/3), that is, 65km/hr. This act of division can be seen to be valid by using the double number line. Again this use of the double number line is in 4 steps.

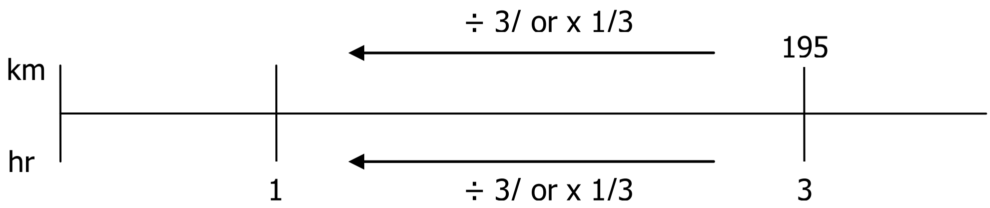
Step 1 Determine in the rate which one is the "per". For example: for \$/L the "per" is litres. For km/hr, the "per" is hours. This is the bottom of the line and the attribute is the top of the line. For example: "The car travelled 195km in 3 hours. What was the speed?" Speed is km/hr so hour on the bottom and km on the top.



Step 2 Put in the numbers- note where the ? is on this example. Note that the hr is 1 in a rate "per hour".



Step 3 Determine Multiplication or division



Step 4 Calculate the unknown: ? = 195 ÷ 3 = 65 km/hr

(2) Complete the following:

i) The cost of mobile phone calls was 75c per 30 seconds. What was the rate in terms of cents/secs?

ii) The petrol cost for 45L was \$57.30. What was the rate in terms of \$/L?

iii) The tap delivered 47L of water in 5 hrs. What was the rate in terms of L/HR?

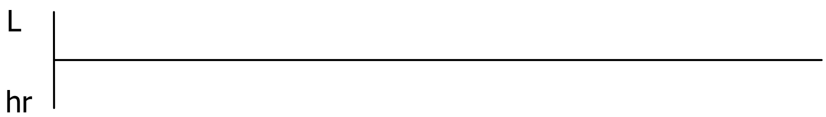
iv) The painter covered 20m² of wall with 8L of paint. What is the painting rate in terms of m²/L?

2.3 Applying rate calculation to speed and fuel consumption

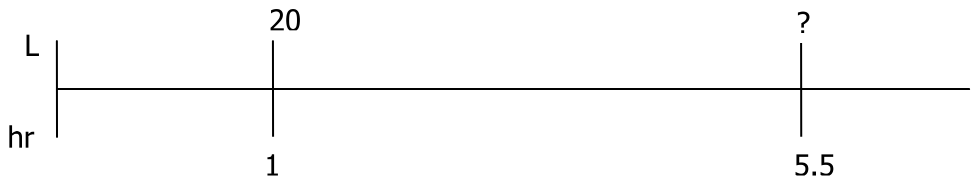
(1) Rate calculations for speed and fuel consumption can be calculated using the double number line in 4 steps as follows

(i) "The boat uses 20L/hr at normal speed. How much fuel for 5 ½ hours?

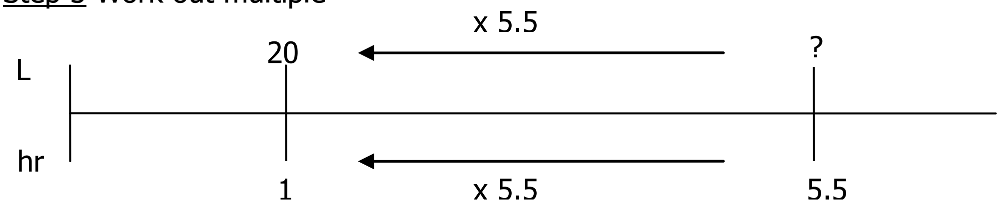
Step 1 Set up number line



Step 2 Put in numbers (including unknown)



Step 3 Work out multiple

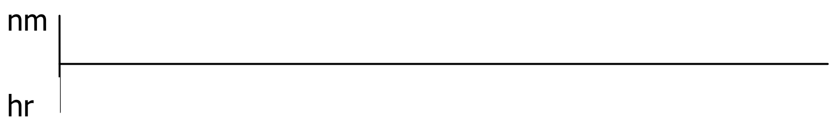


Step 4 Calculate: ? = 20 x 5.5 = 110 Litres

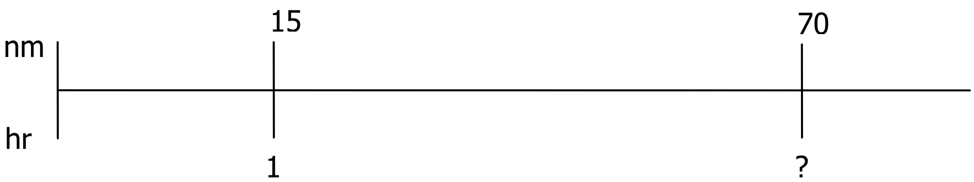
(2) Sometimes two lines (or a triple number line) can be used together

(i) "The boat travels at 15 knots (15nm/hr). How many hours to travel 70nm?"

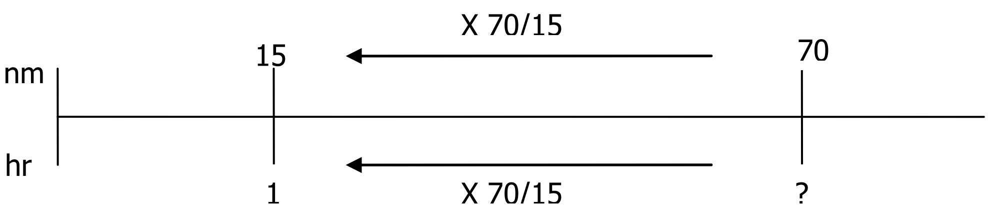
Step 1



Step 2



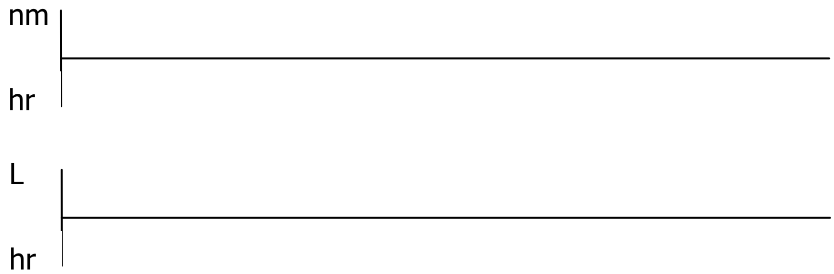
Step 3



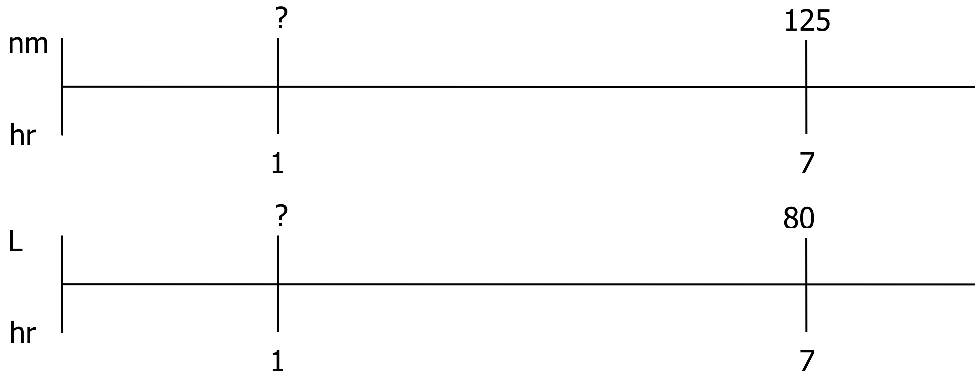
Step 4 ? = 70 ÷ 1 = 4 2/3 or 4.67 hours

(ii) "The boat took 7 hours to travel 125 nm and used 80L of fuel. How fast and what was the fuel consumption rate?

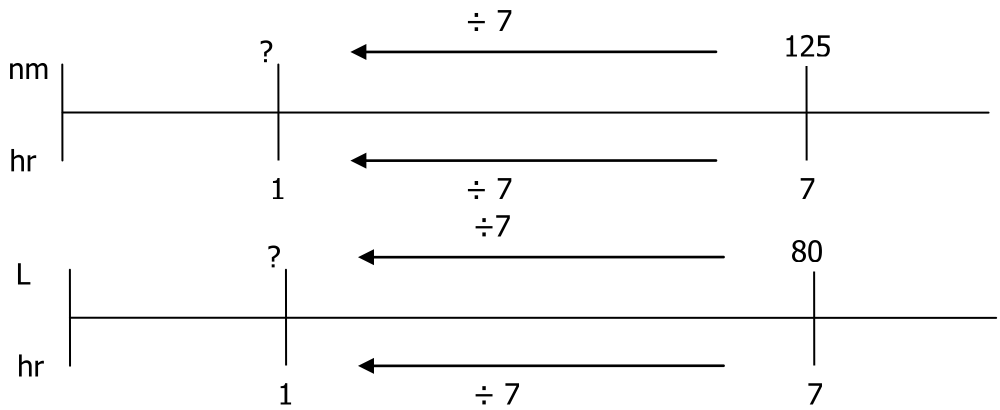
Step 1



Step 2



Step 3



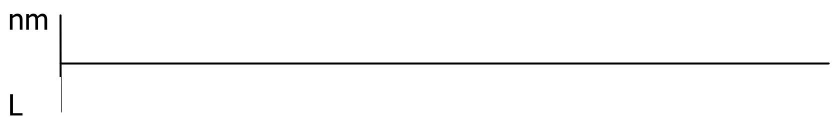
Step 4 Speed (?) = 125 ÷ 7 = 18.9 nm/hr (knots)

Fuel consumption = 80 ÷ 7 = 11.4 L/hr

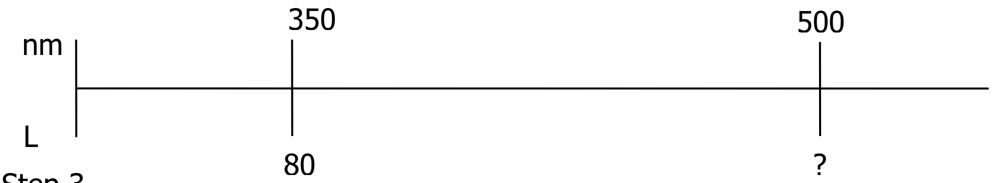
(3) Combination problems can be done without going through rate.

(i) Example: "Boat uses 80L to travel 350 nm. How much petrol would it need to travel 500nm?"

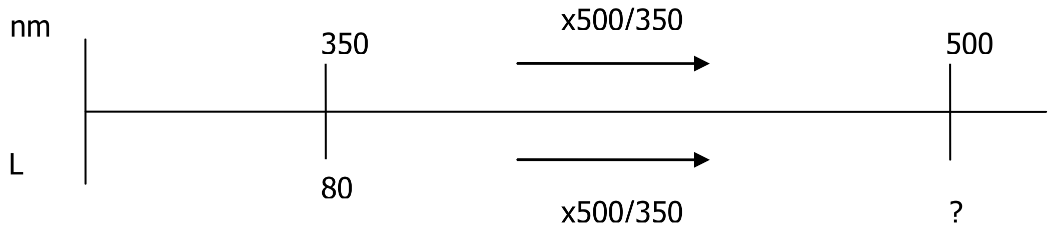
Step 1



Step 2



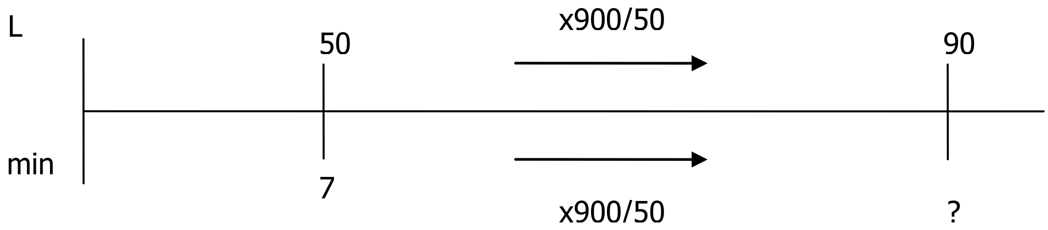
Step 3



Step 4 ? = 80 x 500 ÷ 350 = 114.3L

(ii) Example: "Refuelling hose delivers 50L in 7 minutes. How long to fill a 900L tank?

Steps 1 - 3



Step 4 Time = 7 x 900 ÷ 50 = 126 mins = 2 hrs & 6 mins

(4) Complete the following:

(i) A boat travels 87 nm in 7 hours. What speed was it doing in knots (nm/hr)?

(ii) A boat travels at 16nm/h. How many hours to do 120nm?

(iii) A boat travels 68 nm in 5 hours. How far can it go in a day?

(iv) A ship uses 60L/hr. How much fuel for steaming for 7 ¼ hours?

(v) A ship uses 5000L after 16 hours. What is its fuel consumption rate?

(vi) A ship steams at 23 knots and uses 75L/hr. How far can it go on a 1000L tank

(vii) A ship steams at 30 knots for 11 hours and uses 150L of fuel. How long will it take to go 1000nm and how much fuel?

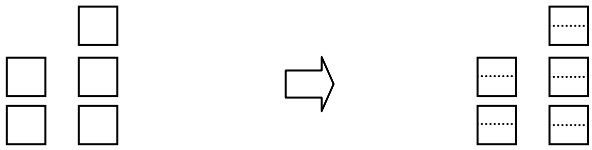
3. USING RATIO TO DETERMINE TIDES

3.1 Understanding Proportion

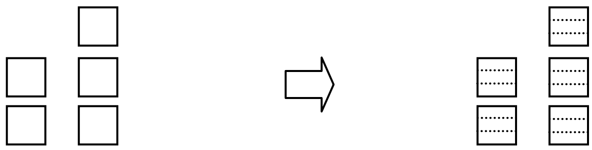
There are 3 ways of dealing with proportion.

(1) *Cutting Squares*

Take diagram of 2:3 and cut squares in half. The ratio is now 4:6



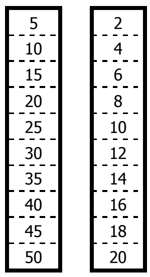
Cut squares into 3 parts, then 2:3 = 6:9



(2) *Proportion Sticks*

Construct proportion sticks (see page 4).

A ratio 2:5 can be shown by placing the 2 stick beside the 5 stick.



All the equivalent ratios can then be seen: 2:5 = 4:10 = 6:15 = ______

(3) *Noticing Multiples*

The two ratios 2:3 and 4:6 are in proportion. Notice the multiples. The two ratios 2:3 and 14:21 are also in ratio. Notice their multiples.

x2 x7

2 : 3 = 4 : 6 2 : 3 = 14 : 21

x2 x7

We can now work out proportions by keeping multiples the same.

x3 x8

3 : 7 = 9 : ? ? : 7 = 24 : 56

x3 x8

(4) *Complete the following*

(i) Construct and cut squares to show

a) 1:2 = 2:4 = 3:6

b) 4:1 = 8:2 = 12:3

c) 3:5 = 6:10 = 9:15

(ii) Take ratio sticks as required and complete proportions

a) 3:7 = 9:__ = __:28 = 27:__

b) 6:2 = 36:__ = __:14

c) 8:5 = 40:__ = __:30

(iii) Complete the following

(a)

x2

4 : 5 = 8 :____

x2

(b)

x4

8 : 5 =____: 20

x4

(c)

x7

1 : 4 =____: 28

x7

(d)

x3

____: 4 = 9: 12

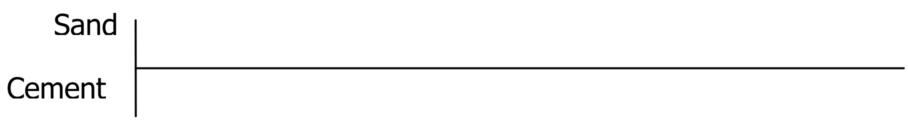
x3

3.2 Solving Problems using double number line

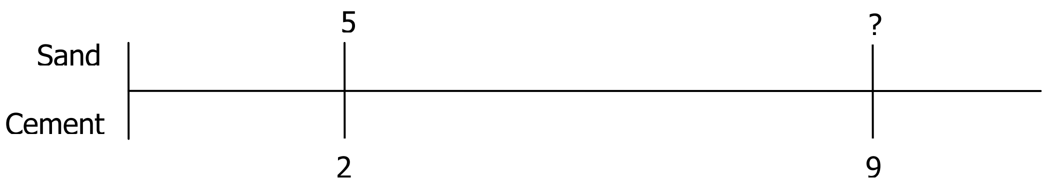
(1) Similar to rate, ratio (and proportion) problems can be easily added with the double number line.

(i) Example "sand and cement are in ratio 5:2. How much sand for 9 tonnes of cement?

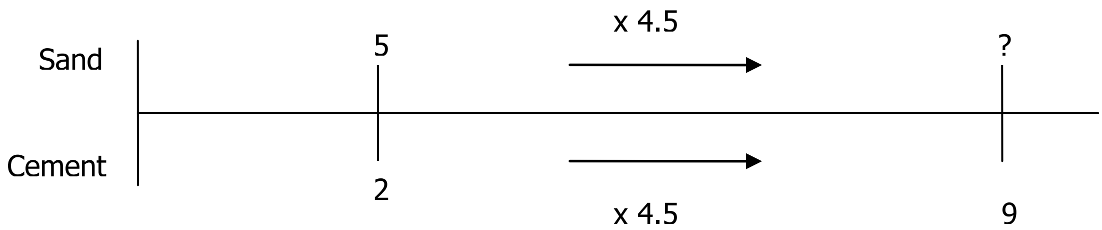
Step 1



Step 2



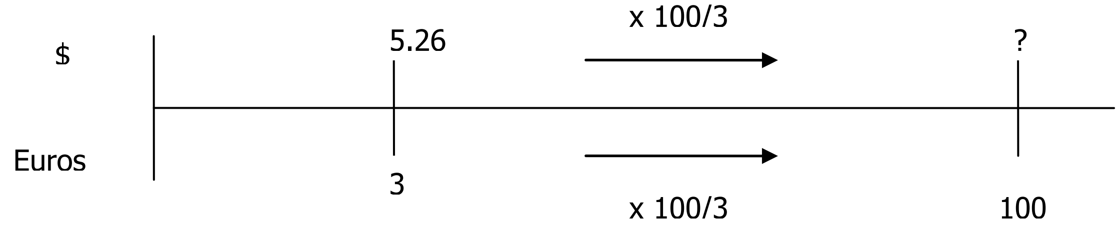
Step 3



<u>Step 4</u> ? (Sand) = 5 x 4.5 = 22.5 tonne

(ii) Example: "I received 3 Euro's for $5.26. How many dollars for 100 Euros?

<u>Steps 1, 2 & 3</u>



<u>Step 4</u> ? = 5.26 x 100 ÷ = $175.33

(2) Complete the following activities. Use the double number line to solve

(i) Cordial and water is in ratio 2:7. How much cordial for 35 litres of water? How much mixture does this make?

(ii) Boy to man is 3:5 in height. The boy is 102cm high. How high is the man?

(iii) Sand to cement is 5:2. How much cement for 35 tonne of sand?

(iv) Flour to butter is 4:7. How much flour for 50kg of butter?

3.3 Tide differences and tidal sine curve

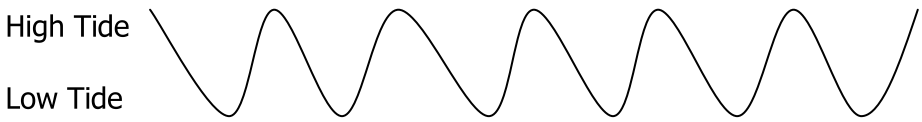
(1) Subtractions has three meanings- (a) takeaway (I had $5, Jim took $2, how much left?), (b) missing addend (I have $2, how much more money to make $5?) and (c) comparison (I have $2; Jim had $5, what is the difference between the money we have?).

(2) Subtraction computation can be done by three strategies: (a) separations (52-35 is 5 tens – 3 tens and 2 ones – 5 ones which we make 4 tens-3ones and 12 ones-5 ones); (b) sequencing 952-35) is two steps: 52-30=22 and 22-5 = 17); and (c) compensation (52-35 is the same as 57-40).

(3) The easiest method for decimal depth and time is often a combination of missing addend and sequencing- e.g., how far from 0.9m to 3.3m (see [A]) and how far from 2.40pm to 11.28am (see [B]) – this is best taught with a number line and has its own algorithm (although it is good for mental computation as well):

[A]	0.90 →	0.10	[B]	2.40 →	0.20
	1.00 →	2.00		3.00 →	8.00
	3.00 →	0.30		11.00 →	0.28
	3.30 →	2.40		11.28 →	8.48

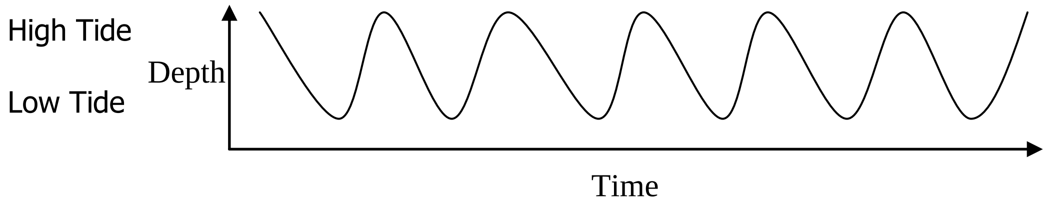
(4) Look at tides and what affects them and study the sine nature of the curve:



Look at how a pen running along the curve goes up and down slow-fast-slow.

(5) Use computer techniques to study how the curve operates.

(6) Look at how time and depth operate on the curve on different axis:



3.4 Using proportional thinking (& vertical number lines)

(1) Use the above sine curve to look at time – depth relationship for tides – this is based on "1/6 rule of thumb" because tides are approximately 6 hours long. We show this by using a vertical number line.

Depth	Time
High	High
11/12	5/6
9/12	4/6
6/12	3/6
3/12	2/6
1/12	1/6 (1h)
Low	Low

(2) So across the six hours, the tide varies slowly and quickly. Consider low is 0.7m and high is 4.3. This gives 3.6m of tide. Since we deal in 12ths for depth, this is 0.3m for each 12th. Consider it starts at 2.10am and finishes at 8.10am.

Depth	Time
4.3 m	8.10 am
4.0 m	7.10 am
3.4 m	6.10 am
2.5 m	5.10 am
1.6 m	4.10 am
1.0 m	3.10 am
0.7 m	2.10 am

(3) Then we can use the double number line to relate the 2 sides. For example "when would a boat of depth 3m be able to arrive at the mooring?" Putting a line across at 2.5m, we see that on the other side it is just over halfway through the 4th hour which is just over half way between 5.10 and 6.10 or just over 5.40. So arriving at around 5.50 so be ok.

Depth	Time
4.3 m	8.10 am
4.0 m	7.10 am
3.4 m	6.10 am
2.5 m	5.10 am
1.6 m	4.10 am
1.0 m	3.10 am
0.7 m	2.10 am

3.5 Calculating Tide Times

(1) Let's do another example. "A boat with depth 3.1m has to leave before it is grounded. The tide is high but it reduces from 4.8m to 0.6m in 6 hours between 3.20pm and 9.20pm. When does the boat have to leave by?"

Depth	Time
0.60 m	9.20 pm
0.95 m	8.20 pm
1.65 m	7.20 pm
2.70 m	6.20 pm
3.75 m	5.20 pm
4.45 m	4.20 pm
4.80 m	3.20 pm

The tide difference is 4.2m. 1/12 of this is 0.35m. So tides and depths as on right using 1/12, 2/12, 3/12, 3/12, 2/12, 1/12 approximations. 3.1m is water 2/3 along the 4th hour. So the boat will need to leave by 6.00pm.

(2) Complete the following

(i) Find the departure time for the following:

a) Depth 3.5m

Tide 1.6m to 6.4m

Time High to low 6.50pm to 12.50am

b) Depth 2m

Tide 0.8m to 2.6m

Time Low to high 10.05am to 4.05pm

(ii) Find the arrival time for the following:

a) Depth 6m

Tide 4.2m to 7.8m

Time High to low 6am to 12pm

b) Depth 5.5m

Tide 1.8m to 6m

Time High to low 11.15am to 5.15pm