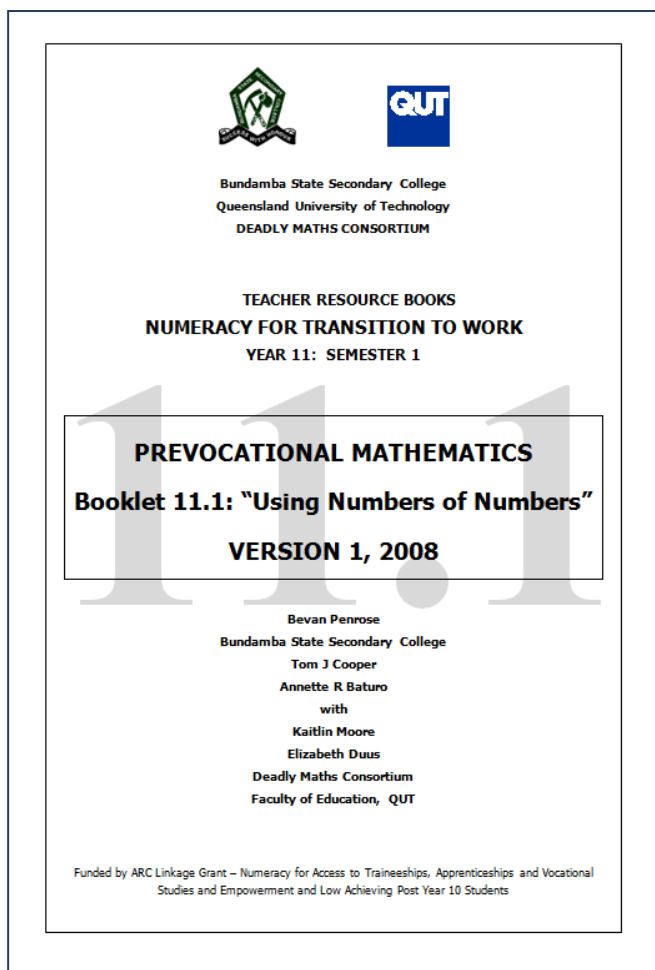




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Numeracy for Transition to Work
Year 11 Prevocational Mathematics
Booklet 11.1: “Using Numbers of Numbers”
Number, decimals, fractions and problem solving



YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called “Deadly Maths”.

“YuMi” is a Torres Strait Islander word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at <http://ydc.qut.edu.au> and staff can be contacted at ydc@qut.edu.au.

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This booklet was developed as part of a project which ran from 2005–2008 and was funded by an Australian Research Council Linkage grant, LP0455667: *Numeracy for access to traineeships, apprenticeships, and vocational studies and empowerment and post Year 10 students*.

Bundamba State Secondary College &
Queensland University of Technology

TEACHER RESOURCE BOOKLETS
NUMERACY FOR TRANSITION TO WORK

YEAR 11 PREVOCATIONAL MATHEMATICS
BOOKLET 11.1: "USING NUMBERS OF NUMBERS"
VERSION 1: 2008

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DEADLY MATHEMATICS VET

Deadly Maths VET was the name given to the materials produced to support the teaching of numeracy to vocational education and training students, particularly those from Indigenous backgrounds. These booklets were produced by the *Deadly Maths Consortium* at Queensland University of Technology (QUT) but also involving a researcher from Nathan Campus of Griffith University.

At the time of the production of this booklet, Deadly Maths VET was producing materials as part of an ARC-funded Linkage grant LP0455667 (12 booklets on Years 11 and 12 Prevocational Mathematics course and 2 booklets on pesticide training) and ASISTM-funded 2008 grant (3 booklets on construction; 3 booklets on engineering; 3 booklets on marine; 1 booklet on retail; and 2 booklets and a series of virtual materials on basic mathematics).

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THE PREVOCATIONAL MATHEMATICS BOOKLETS

In 2005 to 2008 researchers Annette Baturo and Tom Cooper from the Deadly Maths Consortium received an ARC Linkage grant (LP0455667) to study mathematics learning of Year 11 and 12 and adult students undertaking vocational education and training (VET) courses who had low achievement in mathematics. The title of this ARC Linkage project was *Numeracy for access to traineeships and apprenticeships, vocational studies, empowerment and low-achieving post Year 10 students*. The project studied better ways to teach mathematics to VET students at Bundamba State Secondary College, Metropolitan Institute of TAFE (Moreton Campus), Gold Coast Institute of TAFE (Ridgeway Campus), Tagai College (Thursday Island Campus), and the Open Learning Institute of TAFE.

As part of the study, the Deadly Maths research team developed booklets and other resources to be trialled by VET students and teachers. The project's activity with Bundamba State Secondary College focused on the Year 11 and 12 Prevocational Mathematics subjects taught at that college to less able mathematics students. The project used a series of intervention case studies to research learning. As part of this, the following 12 Prevocational Mathematics resource booklets were produced (6 for Year 11 and 6 for Year 12). The booklets are numbered 11.1 to 11.6 and 12.1 to 12.6.

- 11.1 – “Using Numbers of Numbers” – Yr 11 prevocational maths booklet: Number, decimals, fractions and problem solving.
- 11.2 – “The Big Day Out” – Yr 11 prevocational maths booklet: Number, tables, budgeting, algebra and problem solving.
- 11.3 – “Rating our World” – Yr 11 prevocational maths booklet: Rate, area and volume activities and problems.
- 11.4 – “Exchange Student” – Yr 11 prevocational maths booklet: Operations, discounts, tables, metric conversion and best buys.
- 11.5 – “Planning a Roster” – Yr 11 prevocational maths booklet: Tables, 24-hour time, percentages and computation strategies.
- 11.6 – “The Man from Hungary” – Yr 11 prevocational maths booklet: Time relationships, time calculations, timetables and efficient scheduling.
- 12.1 – “Beating the Drought” – Yr 12 prevocational maths booklet: Fractions, probability, graphing and data.
- 12.2 – “Monopoly” – Yr 12 prevocational maths booklet: Fractions, probability, game strategies, property finance, graphs and tables.
- 12.3 – “How tall is the Criminal?” – Yr 12 prevocational maths booklet: Multiplicative structure, ratio and proportion, problem solving.
- 12.4 – “Design a Kitchen” – Yr 12 prevocational maths booklet: Visual imagery, percent, rate, ratio, perimeter, area and volume.
- 12.5 – “Healthy Eating” – Yr 12 prevocational maths booklet: Data collection and analysis, tables and graphing (line and histograms).
- 12.6 – “Rocking around the World” – Yr 12 prevocational maths booklet: Time and angle, time operations and problem-solving strategies.

OVERVIEW

1. Theoretical position

The Bundamba Prevocational Mathematics booklets are based on the notion of Renzulli (1977) that mathematics ideas should be developed through three stages.

Stage 1: Motivate the students – pick an idea that will interest the students and will assist them to engage with mathematics.

Stage 2: Provide prerequisite skills – list and then teach all necessary mathematics ideas that need to be used to undertake the motivating idea.

Stage 3: Culminating task – end the teaching sequence by setting students an open-ended investigation to explore.

These booklets use Stage 3 as an assessment item and so we have added an assessment rubric whereby the culminating task can be used to check the knowledge held by the student.

The booklets combine two approaches to teaching:

- (1) structural activities that lead to the discovery and abstraction of mathematical concepts, processes, strategies and procedures; and
- (2) rich-style tasks which allow students an opportunity to solve problems and build their own personal solution.

2. Mathematics for this booklet

This first learning unit in Year 11 Prevocational mathematics focuses on students completing a variety of numerical activities that involve numerical procedures and common and decimal fractions. The prerequisite activities therefore include: (a) identifying and following numerical procedures to meet numerical outcomes; (b) understanding common fractions and the five meanings for these; (c) understanding decimal numbers and their relation to common fractions; (d) understanding how to interpret word problems; and (e) understanding how to tackle one-step and multi-step problems.

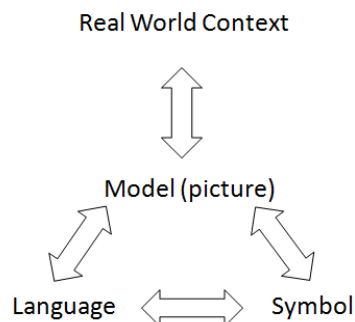
The basis of this booklet is as follows:

- (1) Many calculations are based on algorithmic procedures – a limited set of directions that reach an exact conclusion. Many activities in living and working require skill in being able to identify and follow directions.
- (2) Numbers in life and work are most likely to be decimal numbers. These relate to common fractions through division – the common fraction $\frac{3}{4}$ is the same as $3 \div 4$ which is 0.75. Interestingly some fractions are such that their decimal counterpart goes on forever, a recurring pattern of numbers – for example $\frac{2}{3}$ or $2 \div 3$ is 0.666... on forever. We write it as $0.\dot{6}$ or $0.\bar{6}$.
- (3) Problems in the world involving these numbers often require more than one step. For example "how many posts at \$11.45 each to fence 40 m if a post has to be put every 5 m". This requires a division step: $40 \div 5 = 8$. So 9 posts (need one at each end), and a multiplication step to get the cost: $9 \times 11.45 = \$103.05$. This requires a combination of two skills: (a) breaking the problem into steps, and (b) determining for each step the numbers and operations for the calculator.

3. Pedagogy

The pedagogy for the culminating task is to: (a) interest the students in the situations so they are engaged in the task, (b) provide them with all they need mathematically to gather information and computer totals, and (c) let them develop their solution as they see fit.

The pedagogy for the prerequisite skills is to develop mental models (pictures in the mind) and connect all representations for the mathematics concepts, processes, strategies and procedures that are needed to tackle the culminating task. Thus instruction is based on the so-called Rathmell Triangle (Payne & Rathmell, 1978) on the right:



The prerequisite pedagogy also involves three major generic strategies:

- (1) Flexibility – trying to ensure students understand things in a variety of ways (e.g., discount, reduction, etc.; %, part per 100, decimal hundredths, fraction out of 100, and so on.)
- (2) Reversing – trying to teach in all directions (e.g., real-world situation to best buy; best buy to real-world situation).
- (3) Generalising – trying to teach things in the most general way (going beyond the needs of a task).

Particular strategies for tackling some of the preliminary activities will be provided at the start of the activities.

4. How to use this booklet

The major focus of the unit is the culminating task. The preliminary activities are only suggestions for prior work if you think your students require this work before they begin the culminating task. Therefore:

- (1) use the culminating task as the focus of the unit – to motivate engagement;
- (2) look through the preliminary activities and pick and choose things that you believe will be useful for your students – it is not necessary to do everything or to do it in the order that it appears in this booklet (although there is a logic to the order);
- (3) do these activities as a lead in to the culminating activity; and
- (4) try to organise things so that the students can do the culminating activity as an assessment of their abilities to do mathematics.

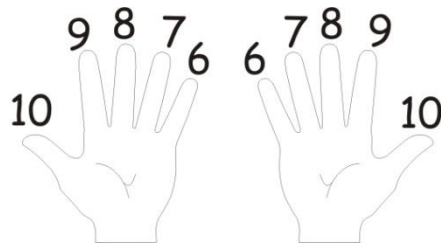
The preliminary activities are in sections and there are real activities at the end of each section. The earlier parts of each section simply explain the ideas/models/pictures in the mind that are being attempted. Although how they are presented gives some hints on pedagogy, you will have to determine your own way to teach these.

PRELIMINARY ACTIVITIES

1. Numerical procedures

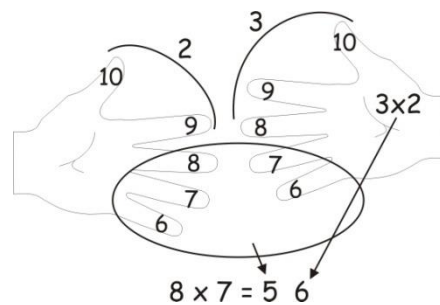
The culminating task requires students to be able to understand and follow procedures with regard to number and operations. Some other examples would provide students with some practice in this area.

1.1. Russian Peasant multiplication facts



To work out a multiplication basic fact, such as 8×7 , above the $5 \times$ tables, do the following:

- (1) Place the fingers together (the 8 and the 7)

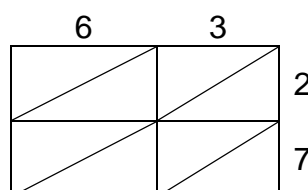


- (2) Add the fingers touching and the fingers below these fingers (the 6 and 7 on the right and the 6, 7 and 8 on the left): $2 + 3 = 5$, this gives the tens, that is 50.
- (3) Multiply the fingers above those touching (this is 3 on the right and 2 on the left): $3 \times 2 = 6$, this gives the ones, that is 6.
- (4) Combining both numbers gives the answer 56.

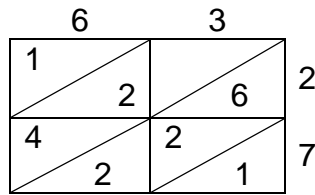
1.2. Russian Peasant multiplication method

To multiply example 63×27 , do the following:

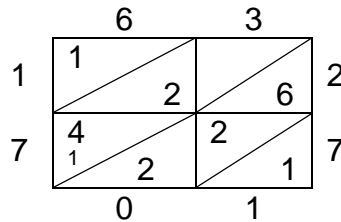
- (1) Construct a lattice and put numbers along top and side.



- (2) Multiply each set of numbers – above the diagonal for tens, below the diagonal for ones

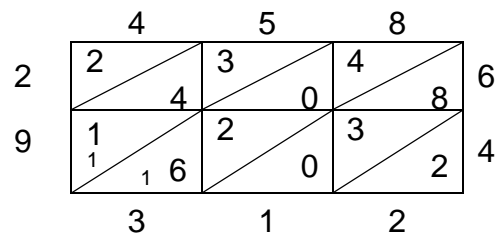
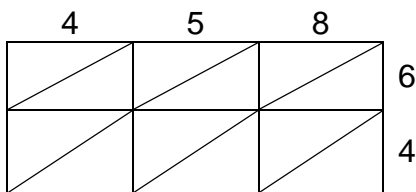


- (3) Add along the diagonals, carrying to the left



The answer is 1,701

Here is another example 458×64



Answer: 29,312

1.3. Numerical procedure activities

- (1) Use the Russian Peasant multiplication facts to calculate:
- (a) 9×9 (b) 6×7 (c) 9×7
- (2) Use the lattice method to calculate:
- (a) 36×82 (b) 276×68
- (3) Search history books and the internet for other examples of number and calculation procedures. Two interesting ones are: (a) the scratch method for operations (used in 17th and 18th century), and (b) the Egyptian technique for writing fractions.

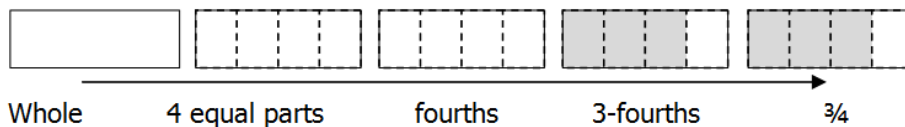
2. Common fractions

The second aspect covered in the culminating task was rational number – common fractions and decimal fractions. In this section we look at common fractions – particularly how common fractions can become decimal fractions.

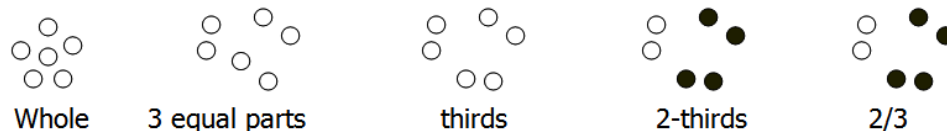
2.1. Meanings of common fractions

There are five meanings of common fractions.

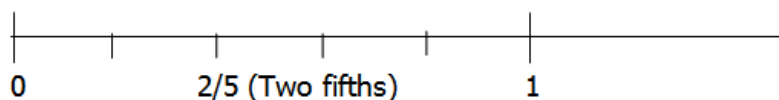
- (1) Fractions as part of a whole. This meaning involves five steps:
- identifying a whole (and ensuring it is held throughout activity),
 - partitioning the whole into equal parts,
 - naming each part by the number of parts (e.g. 2-halves, 3-thirds, 4-fourths),
 - completing the name by identifying, shading, removing a number of the parts, and
 - introducing symbols as parts considered over total number of parts. For example:



- (2) Fractions as part of a group: This follows the same 5 steps as “part of a whole”.
- Combining (unitising) the group into a whole (and ensuring it is held throughout activity),
 - partitioning the whole into equal parts,
 - naming each part by the number of parts (2-halves, 3-thirds, 4-fourths, 5-fifths and so on),
 - completing the name by considering (or shading or removing) a number of the parts, and
 - introducing symbols as parts considered over total number of parts. For example:



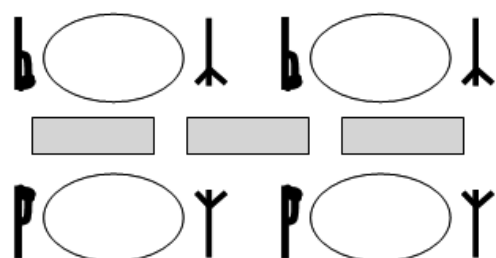
- (3) Fractions as rank or a single point on the number line: No matter what two numbers make up their fraction symbols, each fraction is a single point on a number line. For example:



Once again, the whole (0 to 1) is defined, and partitioned into 5 equal length sections. This makes the second partition point two-fifths or $2/5$ as there are 5 equal lengths and we have moved to the end of the second length.

- (4) Fractions as division: Division can be understood as sharing, so $3 \div 4$ is 3 shared amongst 4. So let us share 3, say, cakes amongst 4 people:

Make 3 paper cakes:



- (a) Share these “cakes” amongst 4 people
- (b) How much did each person get?
- (c) What does this mean? For $3 \div 4$ and $\frac{3}{4}$?

Sharing 3 cakes amongst 4 people requires each cake to be shared independently so that each person gets $\frac{1}{4}$ of the cake. Overall, sharing the 3 cakes means $\frac{3}{4}$ to each person. This means that $3 \div 4$ is the same as $\frac{3}{4}$.

- (5) Fraction as multiplier: If we take 12 pencils, then these 12 pencils can be considered in 2 ways,
 - (a) as one whole – then we can find $\frac{5}{6}$ of this by breaking into 6 equal parts (2 pencils in each part) and taking 5 of these parts (10 pencils in all), and
 - (b) as 12 ones – then we can multiply by 5 (gives 60) and divide by 6 (to give 10).

Thus we see that $\frac{5}{6}$ acts as multiplying by 5 and dividing by 6.

2.2. Converting common fractions to decimal fractions

Changing common fractions to decimals requires meaning 4 – division. The fraction $\frac{3}{4}$ is $3 \div 4$ which is 0.75 after the division is completed (use a calculator).

When these divisions are done for a variety of fractions, it will become evident that the divisions give rise to two types of decimal:

- (1) finite – the decimal ends after a certain number of digits, for example, $\frac{3}{8} = 0.375$; and
- (2) repeating – the decimal goes on forever but following a repeating pattern, for example, $\frac{2}{3} = 0.666666666$ for ever, while $\frac{3}{7} = 0.428571428571$ repeating.

Thus, common fractions always give one of these two types of decimals. However, there is another type of decimal – the “going on for ever decimals” that does not repeat. These are not fractions or rational numbers (they cannot be represented by two whole numbers such as $\frac{2}{3}$); they are called irrational numbers and come from special numbers like $\sqrt{2}$.

2.3. Common fraction activities

- (1) Ask students to complete tables that relate representations:

Area model	Set model	Length model	Language	Symbol

- (2) Give one presentation in each row and ask students to fill in the other spaces.
- (3) Use a calculator to determine the following as decimals:
 - (a) $\frac{3}{32}$
 - (b) $\frac{7}{11}$
 - (c) $\frac{37}{79}$
- (4) Use a calculator to determine:
 - (a) $\frac{7}{9}$
 - (b) $\frac{7}{99}$
 - (c) $\frac{7}{999}$
- (5) Find a fraction that has a repeating pattern made up of 5 parts.

3. Decimal fractions

Common and decimal fractions are similar in that they both represent parts of a whole (e.g., fractions) or wholes and parts of a whole (e.g., mixed numbers). Their difference is in their symbols – the notation they use to describe the parts of the whole.

The fraction notation uses two whole numbers and is based on dividing/partitioning the whole into the bottom of the two numbers (e.g., $\frac{2}{3}$ has the whole divided into 3 equal parts). This restricts what fractions can represent. Decimal notation, on the other hand, continues the base 10 number system to the right of the ones into tenths, hundredths, etc. This gives greater flexibility and decimal-fraction notation can describe all that common-fraction notation can, and more – it can also represent irrationals which are not fractions.

3.1. Decimal fraction numeration

Whole numbers follow a place value structure as follows:

Thousands	Hundreds	Tens	Ones

In this structure, the positions increase by $\times 10$ as they move left and they decrease by $\div 10$ as they move right. This structure allows whole numbers to have meaning just as digits by the convention that the right-most digit is in the ones place. For example, for the number

3 6 7 9

The 9 is ones and, therefore, the 7 is tens, the 6 hundreds and the 3 thousands.

If the relationship of moving left $\times 10$ and moving right $\div 10$ is extended past the ones, then new positions emerge – the decimal positions:

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths

As the ones are no longer on the right, a new convention is accepted – which is that a dot follows the ones. Thus, for the number 3 9 6 7.4 5 2, the 7 is the ones and, therefore, the 6 is tens and the 4 is tenths, the 9 is hundreds and the 5 is hundredths, and the 3 is thousands and the 2 is thousandths.

To teach decimals requires that four concepts be considered:

- (1) Counting – all place values count and follow a pattern of 7, 8, 9, 0 with the position on the left increasing by 1 as the 0 appears. This can be taught with calculators or flip charts.
- (2) Place value – the digits in a number reflect their value through their position with ones on the left being larger than those on the right. To teach this concept requires a size material (bundling sticks and MAB for whole numbers and 10×10 grids for decimals) in relation to a position material (place value charts).

- (3) Rank – regardless of their digits, all numbers are one position on a number line with those further down the line being larger than those less further down. To teach this concept requires number lines and 99 boards (including ropes and pegs to put numbers on the rope).
- (4) Multiplicative structure – digits move to left means $\times 10$ and move to right means $\div 10$. This can be taught with calculators, place value charts and digit cards (or moveable strips – the strips are slid across moving the 7 from position to position).

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths
			7		

Note: Although symbols follow a multiplicative place-value structure, the language for decimals follows common fraction language. For example, 3 4.5 7 is 3 tens, 4 ones, 5 tenths and 7 hundredths and is said/written as thirty-four and fifty-seven hundredths.

3.2. Converting decimals to fractions

For finite decimals, the fraction is straightforward. For example, 3.56 is three and fifty-six hundredths, that is, $3 \frac{56}{100}$ or $3 \frac{14}{25}$. For repeating decimals, a more complicated procedure is required.

- (1) notation for repeating decimals is as follows and repeating decimals are changed to fractions as follows:

$$\begin{aligned}
 0.77777\dots & \text{ is written as } 0.\dot{7} \text{ or } 0.\overline{7} \\
 0.68686868\dots & \text{ is written as } 0.\dot{6}\dot{8} \text{ or } 0.\overline{68} \\
 0.562456245624\dots & \text{ is written as } 0.\overline{5624}
 \end{aligned}$$

- (2) multiply the repeating decimal by 10 if it has one number in the repeat, 100 if it has two numbers in the repeat, and so on

$$\begin{aligned}
 0.77777\dots \times 10 &= 7.7777\dots \\
 0.6868\dots \times 100 &= 68.6868\dots \\
 0.56245624 \times 10,000 &= 5624.56245624\dots
 \end{aligned}$$

- (3) subtract the original number

$$\begin{array}{r}
 10 \times 0.\dot{7} = 7.777\dots \\
 - 1 \times 0.\overline{68} = 0.777\dots \\
 \hline
 9 \times 0.\overline{7} = 7
 \end{array}
 \qquad
 \begin{array}{r}
 100 \times 0.\overline{68} = 68.68 \\
 - 1 \times 0.\overline{68} = 0.68 \\
 \hline
 99 \times 0.\overline{68} = 68
 \end{array}$$

- (4) work out the fraction

$$0.\overline{7} = \frac{7}{9} \qquad 0.68 = \frac{68}{99}$$

3.3. Decimal fraction activities

- (1) Ask students to complete tables that relate representations:

Place Value Chart	Language	Symbol

Give one presentation in each row and ask students to fill in the other spaces.

- (2) In France children do not learn fractions; they are supposed to do everything in decimals. Would this work? Let's try:

- (a) Use the fraction calculators to determine

$$\frac{3}{8} + \frac{5}{6}$$

- (b) Change $\frac{3}{8}$ and $\frac{5}{6}$ to decimals and add these two decimals.

- (c) Are the two answers the same?

- (3) Repeat this for $\frac{5}{7} - \frac{3}{11}$ or $\frac{1}{4} \times \frac{3}{7}$ or $\frac{2}{3} \div \frac{3}{4}$

- (a) Are decimal answers the same as fractions? Does this matter? (Try $2 \div 9 \times 9$ on the calculator. Do you get 2? Why?)

- (b) Calculate $\frac{3}{4} \times \frac{6}{11} + \frac{4}{7}$ using decimals only.

- (c) Calculate the fraction for

(a) $0.\overline{3}$

(b) $0.\overline{26}$

(c) $0.\overline{5624}$

4. Problem solving

Solving word problems with fractions requires an understanding of the meanings of operations in relation to fractions, and ability to problem solve when there is more than one step.

4.1. Operations and fractions

In fraction problem situations, numbers from, and not from, fractional parts can be added or subtracted. To understand what to do requires knowing that addition and subtraction situations relate to joining and separating. The best approach is part-part-total:

- (1) joining and separating groups represent addition if the total is unknown, and
- (2) joining and separating groups represent subtraction if a part is unknown (and the total is known).

For the example, "42 people entered the building; 33 were female; how many were men?", the 42 is the total, the 33 is a part, and the unknown (the men) is a part. Using part-part-total, we can see that the operation to solve the word problem is subtraction.

Also in fraction problem situations, fractions of a group have to be found or used to find a number. Multiplication and division relate to these problem situations, and to fractions, in that finding a fraction of a group is multiplying by numerator and dividing by denominator; for example, $\frac{2}{3}$ of 12 is $12 \times \frac{2}{3} = 8$. Conversely, finding a fraction is division; for example, the fraction that is 12 of 16 is found by dividing 12 by 16 and changing to a fraction (e.g., $12 \div 16 = \frac{12}{16} = \frac{3}{4}$).

Fractions of something are multiplication if the product is unknown and division otherwise. For the example, "Some people entered the building; $\frac{2}{3}$ of them were girls; there were 14 girls; how many people?", $\frac{2}{3}$ is a multiplier, the product is 14, while the unknown is the total (e.g., $\frac{2}{3} \times \text{total} = 14$). This means that we have to divide to get the total (i.e., $14 \div \frac{2}{3}$ which is 21).

The four operations can be used in problems together as the following examples show:

- (1) "42 people entered the building; 14 were girls and 12 were women; what fraction were females?"
- (2) "42 people entered the building; $\frac{1}{3}$ were female; how many females?"

For example (i), the women and girls are considered as two groups and joined (this is addition). Then the 26 is divided by 42 to give fraction $\frac{13}{21}$; and (ii), 42 is multiplied by $\frac{1}{3}$ (or multiplied by 1 and divided by 3) to give 14.

4.2. Multi-step problems

Sometimes there are two steps in a problem. For example, there are two steps in the problem: " $\frac{3}{4}$ of the \$48 was paid. How much is left to pay?" The 2 steps are:

- (1) find out how much was paid; and
- (2) work out the amount left.

The first step is multiplication (finding $\frac{3}{4}$ of \$48 = $\frac{3}{4} \times \$48 = 48 \times \frac{3}{4} = \36). The second step is subtraction ($\$48 - \$36 = \$12$).

It is important for students to learn the strategy of breaking a problem into parts, doing each part separately, and then recombining later.

One way to determine what the parts are is to act out problems with fake money and representations of what is being done (e.g., a picture of a car instead of a real car).

4.3. Problem-solving activities

- (1) Circle the operation you would use if you had a calculator.

(a) \$45 of the money came in cash. How much in cheques if the total was \$56?	+ - × ÷
(b) $\frac{3}{4}$ of the \$56 was paid in cash. How much cash?	+ - × ÷
(c) 0.6 of the cost of the radio was paid and this was \$36. How much did the radio cost?	+ - × ÷
(d) I borrowed \$37 from Fred; he had \$16 left. How much did he have to start with?	+ - × ÷

Having trouble? Make up some fake money and act it out with a friend!

(2) Circle the operations you would use in these problems in the order you would do them. You don't need to work out the answers.

- | | |
|--|--|
| (a) He bought the chair for \$60 and sold it for $\frac{2}{3}$ more than he paid. How much did he sell it for? | 1 st step + - × ÷
2 nd step + - × ÷ |
| (b) John gave Fred 0.8 of the \$35 and gave the rest to Joe. How much did Joe get? | 1 st step + - × ÷
2 nd step + - × ÷ |
| (c) 0.4 of the money was taken. The remainder was found to be \$72. How much was taken? | 1 st step + - × ÷
2 nd step + - × ÷ |

CULMINATING TASK

NUMBERS OF NUMBERS

Directions: This first culminating task in this sequence of Year 11 booklets course requires you, the student, to engage with different types of numbers in problems requiring calculations. All work is to be done on your own paper and attached to this assignment sheet.

TASK 1:

Whole Numbers

An interesting way to multiply is called "Russian peasant multiplication".

Consider the product, 43×274 . First, write out two columns of numbers side by side. Each column is headed by one of the two numbers to be multiplied (43 and 274). Make one column by **doubling** each of the previous numbers. Make the other column by **halving** each of the previous numbers, ignoring any remainders and stopping at one. It is better to make the **halving** column under the smaller of the two numbers. The columns should look like the top figure at right. Now look at the bottom figure at right and look at the halving column; find all the even numbers. Cross out all these even numbers and the numbers in the right hand column in the same row. Now add up the right hand column. This will give the correct answer.

43	274
21	548
10	1096
5	2192
2	4384
1	8768
	11782

So $43 \times 274 = 11\,782$

Your teacher will give you 4 sums to do using "Russian peasant multiplication" – **Russian 1, 2, 3 and 4**. Each student will receive 4 different products to do. They will be 2 digit by 2 digit, 2 digit by 3 digit, 2 digit by 4 digit and 3 digit by 4 digit. All calculations must show all the steps in the two columns as above.

43	274
21	548
10	1096
5	2192
2	4384
1	8768
	11782

TASK 2:

Fractions and Decimals

The following fractions have something in common: **$1/4, 1/2, 1/8, 1/5, 5/6, 3/8, 2/5$** .

They are all called "common fractions", and can be written as decimals. The decimal fractions will be of two types:

- (a) terminating decimals such as 0.45
- (b) recurring decimals such as $0.555555\dots$ (which infinitely repeats) = $0.\dot{5}$

If more than one digit recurs, such as $0.262626\dots$, write = $0.\dot{2}6$ or $0.\overline{26}$

To change a common fraction into a decimal, division is required, for example: $5/8 = 5 \div 8 = 0.625$. The calculation is easily done with a calculator. Your teacher can show you how to do it without a calculator.

FD (a): Use a calculator to convert the 7 common fractions above to decimal fractions. Calculators will easily allow common fraction and decimal fraction operations to be done. Your teacher will show you how to use the fraction keys.

FD (b): Choose any 2 of the 7 fractions: $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{5}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{2}{5}$. Use a calculator to add the two fractions. Use a calculator to multiply the two fractions.

FD (c): Do the same calculations with the two fractions written as decimal fractions. Show that the fraction answer and the decimal answer are the same.

TASK 3

Fraction Problems

The following problems all require the use of common fractions and/or decimal fractions to solve them. Use a calculator to solve the problems below. Your work should be set out in an easy to follow manner.

FD1: Half of Queensland's population of 4 850 560 get too much sun. How many Queenslanders get too much sun?

FD2: $\frac{3}{8}$ of the 16 136 people at the concert were under 18. How many people at the concert were under 18?

FD3: Petrol is at 106.54 cents per litre. How much will 55.4 litres of petrol cost? Give your answer in dollars and cents.

FD4: The proportion of men in the 20-45 year age group in China is 0.657. The total population in the age group is 598 500 000. How many women are there in this age group?

FD5: In the elections in Queensland in 2006, approximately $\frac{3}{8}$ of all female voters supported a major party as their first preference, while approximately $\frac{7}{8}$ of all male voters did. If there were 2 539 634 voters and 0.51 were female, calculate the number of females and the number of males who gave their first preference to a major party.

ASSESSMENT RUBRIC

Assessment Form

Question	Knowledge	Application	Explaining
Russian 1			
Russian 2			
Russian 3			
Russian 4			
FD (a)			
FD (b)			
FD (c)			
FD 1			
FD 2			
FD 3			
FD 4			
FD 5			
<i>Overall Grade</i>			

Assessment Criteria

GRADE	KNOWLEDGE	APPLICATION	EXPLAINING
A	Effectively uses given rules to carry out tasks.	Applies rules across all contexts effectively.	Presents detailed solutions logically and clearly.
B	Uses given rules to carry out tasks.	Applies rules across most contexts effectively.	Presents solutions logically and clearly.
C	Uses given rules adequately to carry out tasks.	Applies rules adequately across most contexts.	Presents readable solutions.
D	Uses given rules to carry out parts of tasks.	Applies rules in some contexts adequately.	Presents partial solutions.
E	Did not meet standard D.	Did not meet standard D.	Did not meet standard D.